

# Chapter 1

## The Heat Engine, the Prime Movers, and the Modern Closed Energy Conversion Systems

A heat engine is an apparatus that converts heat in useful work, usually mechanical work. It operates in a cyclical mode and makes use of a fluid: in a single-phase or in two-phase conditions. The engine receives heat from a source at high temperature, transforms a part of it into work and discharges the remaining heat fraction to the environment at a lower temperature.

The engine often consists of various machines, each one devoted to specific tasks (heat exchangers, pumps, turbines, etc.), and its operation mode can be described on the whole by means of the balance equations of energy and entropy. In the real systems, the heat available at high temperature could be the result of chemical or nuclear reactions or could be an outcome of technological processes. The balance equations of mass, energy, and entropy are summarised and discussed in Appendix A with some simple applicative examples.

The heat engine par excellence is the “reciprocating steam engine”: originally designed by Denis Papin (1647–1712) [1, p. 11]. The first operating design of this kind of engine was the “Miner’s Friend” version of Thomas Savery.<sup>1</sup> It is well known that the engine was later perfected by Thomas Newcomen.<sup>2</sup> Then, James Watt<sup>3</sup> introduced the condenser, separating it from the expansion cylinder. The studies of Rankine<sup>4</sup> then led, via the invention of the steam turbine in 1884, to the modern steam cycles.<sup>5</sup>

---

<sup>1</sup>Thomas Savery, 1650–1715. On 2 July 1698 Savery patented an early steam engine “for raising water by force of fire” [2].

<sup>2</sup>Thomas Newcomen, 1664–1729. An English ironmonger who, assisted by John Calley, a plumber, realised the “atmospheric engine.” Ultimately, Newcomen’s machine was capable of working with steam at low pressure, using atmospheric pressure to lower a piston in a cylinder, inside which, the mixing of water with the steam caused the latter to condense.

<sup>3</sup>James Watt, 1736–1819 Scottish.

<sup>4</sup>William John Macquom Rankine, 1820–1872. Scottish engineer and physicist.

<sup>5</sup>The modern steam turbine was invented by Charles Algernon Parsons, 1854–1931, an Anglo-Irish engineer.

By prime movers we usually mean the machines that convert the various primary energies, which are naturally available, into mechanical energy: the steam engine first of all, then the steam turbine, diesel engine, and the gas turbine are amongst the most significant. In fact, they paved the way for the economic development of today, greatly facilitating the transport of goods and people.

A system of energy conversion normally consists of a primary mover and numerous complementary devices. A coal station, for example, requires not just the steam turbine and all the machinery that makes up the primary thermodynamic mover but also devices for heat disposal (the water circulation system, wet or dry cooling towers), for pulverising the coal (coal crushers, coal pulverisers), and its combustion (steam generators, air preheaters, etc.) and for removing the ash and treating the combustion gases before they are discharged into the atmosphere (a stack, fabric filters, electrostatic precipitators, flue gas desulfurisation systems, and SCR (selective catalytic reduction) or SNCR (selective non-catalytic reduction) reactors for the removal of nitrogen oxide).

The following sections will consider, firstly, the basic thermodynamic characteristics of heat engines, with a brief mention of refrigerating machines and heat pumps: a useful complement to thermodynamic engines, especially if combined with renewable energies and thermal storage systems. Then, consideration will be given to energy conversion systems, followed by descriptions of the traditional and principal prime movers that operate with closed thermodynamic cycles.

## 1.1 The Thermodynamic Characteristics of the Heat Engine<sup>6</sup>

In Fig. 1.1,  $R_H$  and  $R_C$  represent two heat reservoirs<sup>7</sup> at the temperatures  $T_H$  and  $T_0$ , respectively. The engine between the two reservoirs, operating in cyclical mode, exchanges the thermal powers  $\dot{Q}_{in}$  and  $\dot{Q}_{out}$  just with the two reservoirs and produces mechanical power  $\dot{W}$ . For the engine, we can write the power and entropy balances. In stationary conditions:

$$\dot{Q}_{in} - \dot{Q}_{out} - \dot{W} = 0 \quad (1.1a)$$

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_G = 0 \quad (1.1b)$$

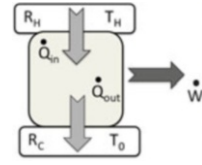
$\dot{S}_{in}$  is the entropy associated with the thermal power  $\dot{Q}_{in}$ , which flows from the reservoir  $R_H$  towards the engine. Likewise,  $\dot{S}_{out}$  is the entropy associated with the thermal power  $\dot{Q}_{out}$ , flowing from the engine towards the cold reservoir  $R_C$ .

---

<sup>6</sup>The results in this and in the following sections can also be obtained by the rigorous application of the energy and entropy balances, as presented and discussed in Appendix B.

<sup>7</sup>A thermal reservoir is an ideal system which constantly maintains a stable state of equilibrium. A thermal reservoir is such in as much that any exchange of heat energy will not affect the temperature, which remains constant [3, p. 106].

**Fig. 1.1** A schematic representation of a heat engine operating between two heat reservoirs  $R_H$  and  $R_C$  at temperatures  $T_H$  and  $T_0$ , respectively



The term  $\dot{S}_G$  represents the entropy generated in the engine in the time unit and during the interaction between the engine and its surrounding systems. Thus, in the case of Fig. 1.1:

$$\dot{S}_{in} = \frac{\dot{Q}_{in}}{T_H}$$

$$\dot{S}_{out} = \frac{\dot{Q}_{out}}{T_0}$$

and

$$\begin{aligned} \frac{\dot{Q}_{out}}{\dot{Q}_{in}} &= \frac{T_0}{T_H} \frac{\dot{S}_{out}}{\dot{S}_{in}} \\ &= \frac{T_0}{T_H} \frac{\dot{S}_{in} + \dot{S}_G}{\dot{S}_{in}} = T_0 \left( \frac{1}{T_H} + \frac{\dot{S}_G}{T_H \dot{S}_{in}} \right) \\ &= T_0 \left( \frac{1}{T_H} + \frac{\dot{S}_G}{\dot{Q}_{in}} \right) \end{aligned}$$

The efficiency whereby the engine converts the thermal power  $\dot{Q}_{in}$  into the mechanical output  $\dot{W}$  is defined as

$$\eta = \frac{\dot{W}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \quad (1.2)$$

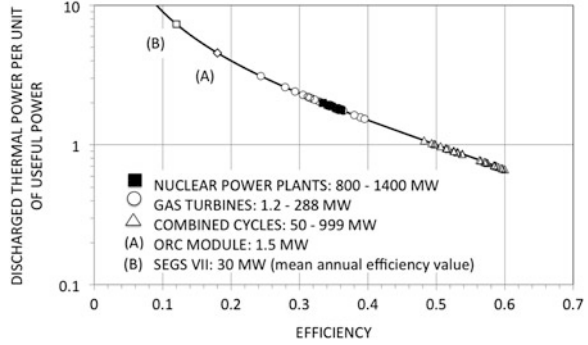
Since

$$\frac{\dot{Q}_{out}}{\dot{Q}_{in}} = T_0 \left( \frac{1}{T_H} + \frac{\dot{S}_G}{\dot{Q}_{in}} \right)$$

the efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - T_0 \left( \frac{1}{T_H} + \frac{\dot{S}_G}{\dot{Q}_{in}} \right) < 1 \quad (1.3)$$

**Fig. 1.2** Discharged thermal power per unit of electrical power as a function of the net efficiency for various engines and energy conversion systems. The efficiency values are relative to the design conditions, except where explicitly indicated



When  $\dot{S}_G = 0$ , the efficiency assumes the maximum value  $\eta_{\max}$ , equal to (in the case considered in Fig. 1.1)

$$\eta_{\max} = 1 - \frac{\dot{Q}_{\text{out, min}}}{\dot{Q}_{\text{in}}} = 1 - \frac{T_0}{T_H} \quad (1.4)$$

The expression (1.4), denominated the efficiency of Carnot,<sup>8</sup> is valid only if there are no irreversibilities in the engine or at the level of heat exchange at extreme operating temperatures. In fact, the term  $\dot{S}_G$  in (1.3) takes into account the various forms of irreversibility (the thermodynamic losses, see Appendix C) that take place inside the engine and at the engine–reservoir interface: external irreversibilities (present at the edges of the system in consideration) and internal irreversibilities (inside the system). Typical irreversibility includes mechanical and fluid friction losses, heat transfer losses and throttling and mixing losses.

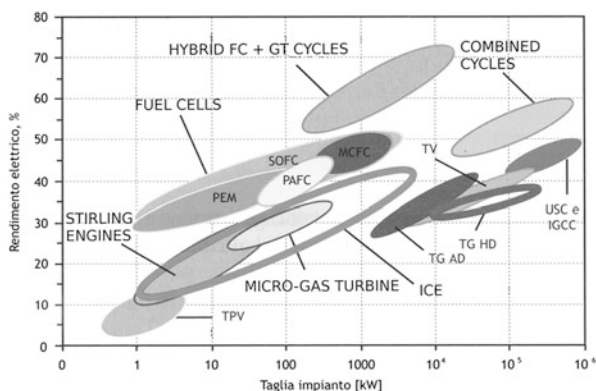
In any case, the thermal power  $\dot{Q}_{\text{out}}$  discharged at the cold sink (usually the environment) per unit of mechanical power output  $\dot{W}$  which is never zero is

$$\frac{\dot{Q}_{\text{out}}}{\dot{W}} = \frac{1}{\eta} - 1 \quad (1.5)$$

The thermal power  $\dot{Q}_{\text{out}}$  is dissipated into the environment via the cooling devices of the engine and by means of any products of the combustion (exhaust gases, ashes, ...). High efficiency values, therefore, not only give (for equal thermal power consumed  $\dot{Q}_{\text{in}}$ ) high useful mechanical power but also lead to reduced environmental pollution: less thermal and chemical pollution.

In Fig. 1.2 the ratio  $\dot{Q}_{\text{out}}/\dot{W}$  as a function of the net efficiency for various engines and energy conversion systems is reported. An efficiency value of 50% means

<sup>8</sup>Nicolas Leonard Sadi Carnot, 1796–1832. French physicist and engineer. Author of fundamental studies on the performance of thermal machines and considered one of the founders of the science of thermodynamics.



**Fig. 1.3** Basic representation of the electric efficiency and of the electric unit power of the systems for energy generation (from author [5])

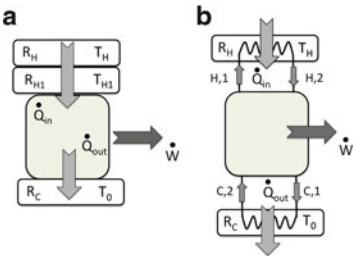
that the heat discharged into the environment equals the useful power. The highest values of efficiency are reached in the engines and systems with the greatest power: efficiencies of around 60 % can be obtained in combined cycles with unitary power over 500 MW. The average annual efficiency values may be significantly lower than the net nominal efficiency. For example, the SEGS power stations, steam-driven solar energy, situated at Kramer Junction in the Mojave Desert (in California), operate with thermodynamic cycles having nominal efficiency of 30–40 % and average annual values of 10–15 % [4].

Figure 1.3 shows the electrical efficiency of various systems of energy generation as a function of power. It shows the heat engines (in closed and open thermodynamic cycles) and the fuel cells (electrochemical engines). Note how a reduction in the power tends to reduce the electrical efficiency, too. By hybrid plants we mean integrated systems of fuel cells at high temperature (MCFC (molten carbonate fuel cells) and SOFC (solid oxide fuel cells)) with gas or steam cycles: these are still in the process of testing and development. If the intention is to favour distributed energy generation (that is, the large-scale diffusion of small-sized thermal engines), given the significant increase in energy performance as power is increased, there is no chance of competing with the electrical efficiency of the large combined cycles. It will be necessary to exploit to the full thermodynamic advantages of the cogeneration of heat and electricity (see Sect. 1.4). In practice, this means it is necessary to fully utilise the energy fluxes of the engine in every working condition.

Some indicative values of specific costs and efficiency with data of emission for the modern thermoelectric plants are in Table 1.1.

**Table 1.1** Approximate values for investment costs, efficiency, and emissions of thermoelectric plants [6]

|                              | Steam power plant | Gas turbine | Combined cycle |
|------------------------------|-------------------|-------------|----------------|
| Fuel                         | Carbon            | Natural gas | Natural gas    |
| Specific cost (euro/kW)      | 1,100–1,200       | 200–250     | 400–500        |
| Efficiency (based on LHV, %) | 43–46             | 35–40       | 55–60          |
| Specific emissions           |                   |             |                |
| NO <sub>x</sub> (g/MWh)      | 300–600           | 400–450     | 100–300        |
| SO <sub>x</sub> (g/MWh)      | 300–600           | <10         | <10            |
| Particulate (g/MWh)          | 30–80             | 0           | 0              |
| CO <sub>2</sub> (kg/MWh)     | 750–850           | 500–600     | 350–400        |



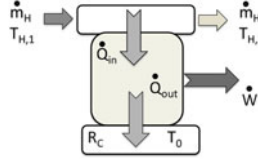
**Fig. 1.4** Heat engines between a heat source at temperature  $T_H$  and a heat sink at temperature  $T_0$  but operating at temperatures different from those of the two reservoirs. **(a)** Engine operating at maximum temperature  $T_{H1} < T_H$ ; **(b)** engine using a working fluid that is heated from temperature  $T_{H1}$  to temperature  $T_{H2} (< T_H)$  and releases residual heat when cooling from  $T_{C1}$  and  $T_{C2} (> T_0)$

**1.1.1 Some Special Combinations of Heat Sources and Heat Engines**

Let's now consider various different systems of heat engines, heat sources, and heat sinks. In particular, we discuss firstly the presence of irreversibility of heat exchange at the level of the hot reservoir (Fig. 1.4a). Then, we analyse the case in which the engine exchanges heat with the hot and with the cold reservoir using a fluid and suitable heat exchangers (Fig. 1.4b) and, lastly, the case in which the heat source is not a reservoir (at constant temperature  $T_H$ ), but a fluid with variable temperature (Fig. 1.5).

If the engine (see Fig. 1.4a), which is completely reversible, receives heat from a hot reservoir at temperature  $T_H$ , but begins its conversion from a temperature  $T_{H1} < T_H$ , then this gives rise to an irreversibility of heat exchange equal to

$$\dot{S}_G = \dot{S}_{G,H} = \frac{\dot{Q}_{in}}{T_{H1}} - \frac{\dot{Q}_{in}}{T_H} = \dot{Q}_{in} \left( \frac{1}{T_{H1}} - \frac{1}{T_H} \right) \tag{1.6}$$



**Fig. 1.5** Scheme of a heat engine receiving heat from a sensible heat source (with temperature variable between  $T_{H,i}$  and  $T_{H,2}$ ) and discharging heat into the cold reservoir at temperature  $T_0$

with a conversion efficiency that is consequently equal to

$$\eta = 1 - \frac{T_0}{T_H} - T_0 \left( \frac{1}{T_{H1}} - \frac{1}{T_H} \right) = 1 - \frac{T_0}{T_{H1}} < 1 - \frac{T_0}{T_H} \quad (1.7)$$

If the engine is still working in a completely reversible mode but with a working fluid that receives heat from the reservoir  $R_H$  and then loses it to reservoir  $R_C$  via two heat exchangers, we get two irreversibilities linked to the heat exchange:  $\dot{S}_{G,H}$  and  $\dot{S}_{G,C}$  (see Fig. 1.4b):

$$\dot{S}_G = \dot{S}_{G,H} + \dot{S}_{G,C} \quad \text{with} \quad (1.8)$$

$$\dot{S}_{G,H} = \dot{m}_H \left[ (S_{H,2} - S_{H,i}) - \frac{H_{H,2} - H_{H,i}}{T_H} \right]$$

$$\dot{S}_{G,C} = \dot{m}_C \left[ \frac{H_{C,1} - H_{C,2}}{T_0} - (S_{C,1} - S_{C,2}) \right]$$

and the efficiency becomes

$$\eta = 1 - \frac{T_0}{T_H} \quad (1.9)$$

$$-T_0 \frac{\dot{m}_H}{\dot{Q}_{in}} \left[ (S_{H,2} - S_{H,i}) - \frac{H_{H,2} - H_{H,i}}{T_H} \right]$$

$$-T_0 \frac{\dot{m}_C}{\dot{Q}_{in}} \left[ \frac{H_{C,1} - H_{C,2}}{T_0} - (S_{C,1} - S_{C,2}) \right]$$

with  $\dot{m}_H$  and  $\dot{m}_C$  representing the mass flows of the two working fluids in the heat exchangers.

A particularly interesting situation occurs when the engine receives heat not from a hot reservoir at temperature  $T_H$  but from a source with variable temperature (for example, from a fluid, gas or liquid, which lowers the temperature from  $T_{H,1}$  to  $T_{H,2}$ ; see Fig. 1.5). In this case, the entropy balance gives (under stationary conditions)

$$\dot{m}_H S_{H,1} - \dot{m}_H S_{H,2} - \frac{\dot{Q}_{out}}{T_0} + \dot{S}_G = 0 \quad \text{or} \quad (1.10)$$

$$\dot{Q}_{out} = T_0 [\dot{m}_H (S_{H,1} - S_{H,2}) + \dot{S}_G]$$

and the efficiency (1.2) becomes

$$\begin{aligned}\eta &= 1 - \frac{T_0}{\dot{Q}_{\text{in}}} [\dot{m}_H (S_{H,1} - S_{H,2}) + \dot{S}_G] \\ &= 1 - T_0 \left[ \frac{\dot{m}_H (S_{H,1} - S_{H,2})}{\dot{Q}_{\text{in}}} + \frac{\dot{S}_G}{\dot{Q}_{\text{in}}} \right]\end{aligned}\quad (1.11)$$

since

$$\dot{Q}_{\text{in}} = \dot{m}_H (H_{H,1} - H_{H,2}) \quad (1.12)$$

finally giving

$$\eta = 1 - T_0 \left( \frac{S_{H,1} - S_{H,2}}{H_{H,1} - H_{H,2}} + \frac{\dot{S}_G}{\dot{Q}_{\text{in}}} \right) = 1 - T_0 \left( \frac{1}{T_H^*} + \frac{\dot{S}_G}{\dot{Q}_{\text{in}}} \right) \quad (1.13)$$

with  $T_H^*$  the equivalent thermodynamic temperature of the heat source and  $\dot{S}_G$  representing, as usual, the sum of all the thermodynamic irreversibilities in the system being considered.

The ratio (1.13) is formally equal to the ratio (1.3) but with  $T_H^*$  in place of the temperature  $T_H$  of the hot reservoir. This latter case is representative of those situations where mechanical (or electric) energy needs to be produced in the presence of sensible heat sources. Typically: the exploitation of geothermal sources at low enthalpy and heat recovery from industrial processes (see Sect. 3.5).

## 1.2 The Heat Pump and the Refrigeration Machines

Figure 1.6 represents a system which, using mechanical power  $\dot{W}$ , has as useful effect the thermal power  $\dot{Q}_{\text{out}}$  discharged at the high-temperature reservoir or the thermal power (refrigerating)  $\dot{Q}_{\text{in}}$  acquired from the reservoir at temperature  $T_C$ .

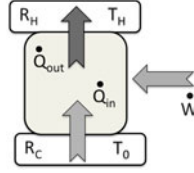
The power and entropy balances, under stationary conditions, give

$$\begin{aligned}\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} + \dot{W} &= 0 \\ \dot{S}_{\text{in}} - \dot{S}_{\text{out}} + \dot{S}_S &= 0\end{aligned}\quad (1.14)$$

Then, the entropy flows associated with the thermal powers in transit are

$$\begin{aligned}\dot{S}_{\text{in}} &= \frac{\dot{Q}_{\text{in}}}{T_0} \\ \dot{S}_{\text{out}} &= \frac{\dot{Q}_{\text{out}}}{T_H}\end{aligned}$$





**Fig. 1.6** Basic scheme of a heat pump or refrigeration cycle that uses mechanical power to produce a useful effect. The useful effect is  $\dot{Q}_{out}$  in the case of a heat pump or  $\dot{Q}_{in}$  in the case of a refrigerating machine

In the case of the heat pump, it is a common practice to resort to a coefficient of merit (the coefficient of performance, COP), defined as the ratio between the thermal power  $\dot{Q}_{out}$  and the mechanical power consumed  $\dot{W}$ :

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_{out}}{\dot{W}} = \frac{1}{1 - \frac{\dot{Q}_{in}}{\dot{Q}_{out}}} \quad (1.15)$$

From the above relationships, we can derive the ratio  $\dot{Q}_{in}/\dot{Q}_{out}$ :

$$\frac{\dot{Q}_{in}}{\dot{Q}_{out}} = \frac{T_0}{T_H} \frac{\dot{S}_{in}}{\dot{S}_{out}} = \frac{T_0}{T_H} \frac{\dot{S}_{out} - \dot{S}_G}{\dot{S}_{out}} = T_0 \left( \frac{1}{T_H} - \frac{\dot{S}_G}{T_H \dot{S}_{out}} \right)$$

or

$$\frac{\dot{Q}_{in}}{\dot{Q}_{out}} = T_0 \left( \frac{1}{T_H} - \frac{\dot{S}_G}{\dot{Q}_{out}} \right)$$

and the COP of the heat pump is

$$\text{COP}_{\text{HP}} = \frac{1}{1 - T_0 \left( \frac{1}{T_H} - \frac{\dot{S}_G}{\dot{Q}_{out}} \right)} \quad (1.16)$$

In the case of the refrigeration machine, the parameter of merit is calculated by the ratio between the refrigerating power  $\dot{Q}_{in}$  and the mechanical power consumed  $\dot{W}$ :

$$\text{COP}_{\text{RM}} = \frac{\dot{Q}_{in}}{\dot{W}} = \frac{1}{\frac{\dot{Q}_{out}}{\dot{Q}_{in}} - 1} \quad (1.17)$$

Just like the previous case, the balances of energy and entropy give the ratio  $\dot{Q}_{out}/\dot{Q}_{in}$ :

$$\frac{\dot{Q}_{out}}{\dot{Q}_{in}} = \frac{T_H}{T_0} \frac{\dot{S}_{out}}{\dot{S}_{in}} = \frac{T_H}{T_0} \frac{\dot{S}_{in} + \dot{S}_G}{\dot{S}_{in}} = T_H \left( \frac{1}{T_0} + \frac{\dot{S}_G}{T_0 \dot{S}_{in}} \right)$$

or

$$\frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = T_H \left( \frac{1}{T_0} + \frac{\dot{S}_G}{\dot{Q}_{\text{in}}} \right)$$

and the COP of the refrigeration machine is

$$\text{COP}_{\text{RM}} = \frac{1}{T_H \left( \frac{1}{T_0} + \frac{\dot{S}_G}{\dot{Q}_{\text{in}}} \right) - 1} \quad (1.18)$$

When  $\dot{S}_G$  is null, the coefficients of performance  $\text{COP}_{\text{HP}}$  and  $\text{COP}_{\text{RM}}$  reach their maximum value:

$$\text{COP}_{\text{HP,max}} = \frac{1}{1 - \frac{T_0}{T_H}} = \frac{T_H}{T_H - T_0} \quad (1.19)$$

$$\text{COP}_{\text{RM,max}} = \frac{1}{\frac{T_H}{T_0} - 1} = \frac{T_0}{T_H - T_0} \quad (1.20)$$

A machine that absorbs a mechanical power  $\dot{W}$  can function both as a heat pump and as a refrigeration machine. In the first case, the useful effect is the thermal power  $\dot{Q}_{\text{out}}$  discharged at temperature  $T_H$ ; in the second case, the useful effect is the thermal power (refrigerating)  $\dot{Q}_{\text{in}}$  absorbed by the heat reservoir at temperature  $T_0$ . The two COPs are connected by the ratio

$$\text{COP}_{\text{HP}} = \frac{\dot{Q}_{\text{out}}}{\dot{W}} = \frac{\dot{Q}_{\text{in}} + \dot{W}}{\dot{W}} = 1 + \text{COP}_{\text{RM}} \quad (1.21)$$

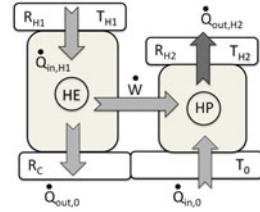
Finally, the ratio (1.18) shows that it is not possible to transfer thermal power  $\dot{Q}_{\text{in}}$  from a reservoir at temperature  $T_0$  towards a reservoir at temperature  $T_H > T_0$  without consuming power. The minimum power that must be consumed can be calculated by the ratio (1.20), and in the real case ( $\dot{S}_G > 0$ ), power consumption will certainly be greater.

## Exercises

**1.1.** We evaluate the thermal power  $\dot{Q}_{\text{out,H2}}$  that can be supplied to a heat reservoir at temperature  $T_{\text{H2}}$  when there is a thermal power  $\dot{Q}_{\text{in,H1}}$  provided by a heat reservoir at temperature  $T_{\text{H1}}$  and with an environment available at temperature  $T_0$ .

In the simplest and most traditional case, the thermal power available at temperature  $T_{\text{H1}}$ , usually obtained by burning fossil fuels, is transferred to the user

**Fig. 1.7** Basic scheme of a prime mover (a heat engine, HE) and a heat pump HP for a heating system



(the heat reservoir at temperature  $T_{H2}$ ) via a boiler. In this case,  $\dot{Q}_{out,H2} = \dot{Q}_{in,H1} \eta_{th}$ , with  $\eta_{th} < 1$  boiler efficiency.

However, referring to Fig. 1.7, we may consider a more complex and sophisticated thermodynamic system, in which the power available  $\dot{Q}_{in,H1}$  is converted into mechanical power by a heat engine HE. The mechanical power  $\dot{W}$  which is generated thereby is used to feed a heat pump HP which, acquiring  $\dot{Q}_{in,0}$  from the environment at temperature  $T_0$ , releases the power  $\dot{Q}_{out,H2}$  to the user, at temperature  $T_{H2}$ . In this case,

$$\dot{Q}_{out,H2} = \dot{Q}_{in,H1} \times \eta_{HE} \times COP_{HP}$$

with  $\eta_{HE}$  representing the efficiency of the engine HE and  $COP_{HP}$  which is the coefficient of performance of the heat pump HP. If  $T_{H1}$  is not too low, if  $T_{H2}$  is not excessively high, and if the system irreversibility losses are not excessive, the product  $\eta_{HE} COP_{HP}$  may be significantly higher of one.

Assuming  $T_{H1} = 800^\circ\text{C}$ ,  $T_{H2} = 35^\circ\text{C}$  and  $T_0 = 0^\circ\text{C}$ , and hypothesising the ideal system ( $\dot{S}_G = 0$ ), we obtain  $\eta_{HE} = 0.745$  and  $COP_{HP} = 8.804$  or the ratio  $\dot{Q}_{out,H2}/\dot{Q}_{in,H1} = 6.56$ .

**1.2.** The conversion efficiency expressed by the relationship (1.3) or, more generally, by the realation (1.13) represents the fraction of the thermal power  $\dot{Q}_{in}$  converted into useful power  $\dot{W}$  when a heat reservoir (normally the environment) is available at temperature  $T_0$ . The efficiency  $\eta$  (efficiency of the first principle<sup>9</sup>) may vary considerably, depending on the application being considered, but it does not represent a priori a valid index of the thermodynamic quality of the conversion process. One parameter, though, that does provide useful information on the thermodynamic quality of the conversion cycle, or “good thermodynamic design” of an engine, is the second principle efficiency  $\eta_{II} = \eta/\eta_{max}$ .

Applying the definition,

$$\eta_{II} = \frac{\eta}{\eta_{max}} = 1 - \frac{T_0 \dot{S}_G}{\dot{Q}_{in} \eta_{max}} = 1 - \frac{T_0 \sum_j \dot{S}_{G,j}}{\dot{Q}_{in} \eta_{max}} = 1 - \sum_j \Delta \eta_{II,j} \quad (1.22)$$

<sup>9</sup>By first principle we mean the First Principle of Thermodynamics.

with  $\sum_j \Delta\eta_{II,j}$  representing the sum of all the losses of thermodynamic availability (see Appendix B and Appendix C) manifested in the conversion system, compared to the maximum efficiency obtainable, given a certain resource of thermal power  $\dot{Q}_{in}$ .

We consider the following cases:

- A traditional steam vapour Rankine cycle for a power station of 300 MW, with superheating at 540 °C at a pressure of 166.7 bar and re-superheating at a pressure of 33.3 bar. Condensation temperature equal to 32.5 °C (condensation pressure of 0.05 bar). Cycle efficiency  $\eta = 0.45$  [7]. The maximum efficiency is  $\eta_{max} = 1 - 305.65/813.15 = 0.624$  and the second-law efficiency  $\eta_{II} = \eta/\eta_{max} = 0.45/0.624 = 0.721$ .
- A steam cycle for a water-pressurised nuclear power station with water temperature exiting the reactor at 329 °C and entering at 292 °C. Efficiency  $\eta = 0.34$ . In this case,  $\eta_{max} \approx 1 - 305.65/583.65 = 0.476$ ,  $\eta_{II} = \eta/\eta_{max} = 0.34/0.476 = 0.714$ .
- An Organic Rankine Cycle (ORC) which recovers heat from a furnace. The heat source consists of the gases produced by combustion and available at 273 °C, which are then cooled to 130 °C. The useful electric power is 0.16 MW. The condensation temperature is 40 °C. Cycle efficiency  $\eta = 0.163$  [9]. In this last case,  $\eta_{max} \approx 1 - 313.15/471.038 = 0.335$ ,  $\eta_{II} = \eta/\eta_{max} = 0.163/0.335 = 0.486$ .

The results show a substantial similarity in the thermodynamic quality of the two high-powered vapour cycles (despite the great difference in their first-law efficiency): in fact, the great power justifies the elevated plant costs, linked to the significant reduction in the irreversibility losses present in the two cycles, thereby achieving the high second-law efficiencies. The small organic fluid engine, despite having good thermodynamic qualities (equal to around 50 % of the ideal cycle), has less drive though and is less sophisticated from a plant engineering point of view.

The relationship (1.22) deserves certain considerations:

- By means of entropy analysis or calculation of the various terms  $\dot{S}_{G,j}$ , different kinds of losses (heat exchange, frictions, chemical reactions, etc.) are made homogeneous and can be compared.
- To evaluate the overall production of entropy, it is fundamental to define clearly the environment with which the system will interact (see also Appendix B and Appendix C). For instance, the preheating of the liquid in a vapour cycle may be more or less reversible according to the characteristics of the external source (a sensible heat source, like geothermal water, for example, with variable temperature, could in principle render the heat exchange reversible).
- If, in the heat transmission processes, the production of entropy is always calculated as the difference between the increase of entropy in the cold fluid and the decrease of entropy in the hot fluid, in the adiabatic transformations (expansion in a turbine, throttling in a valve), the production of entropy

coincides with the increase of entropy in the fluid which is, itself, the subject of transformation.<sup>10</sup>

- The entropic production (directly correlated with a thermodynamic loss) is intrinsically different from the degradation and dissipation of the mechanical work through pressure drops and friction. In fact, if  $\dot{W}$  represents the mechanical power that degrades in thermal power to temperature  $T$ , the consequent entropic production is  $\dot{S}_G = \dot{W}/T$ , which may be significantly inferior at  $\dot{W}$  if temperature  $T$  is high. The reason for this lies in the fact that the heat at temperature  $T$  has its own potential for work production quantifiable via the efficiency of an ideal thermodynamic cycle (Carnot's) between  $T$  and  $T_0$ . This potential is subtracted from the dissipation. It is only at temperature  $T = T_0$  that the potential is cancelled out and the entropic loss coincides with  $\dot{W}$ .

From the above, we can conclude that entropic analysis reveals the definite and unrecoverable losses, where other kinds of losses may be partially recovered.<sup>11</sup>

**1.3.** The conversion efficiency that is normally intended for a system is defined by the relationship (1.2). In the case of systems using fossil fuels, the term  $\dot{Q}_{in}$  is calculated as product of the flow  $\dot{m}_F$  and the LHV (lower heating value) of the fuel:  $\dot{Q}_{in} = \dot{m}_F \times \text{LHV}$ . Other parameters traditionally referred to when using fuels are the specific fuel consumption and the specific heat consumption. Here below, we give their definitions.

- The specific fuel consumption sfc is the quantity (in mass) of fuel per unit of useful work produced. The units of measurement of the parameter sfc are usually kg/kWh :

$$\text{sfc} = \frac{\dot{m}_F}{\dot{W}} = \frac{\dot{m}_F}{\eta \dot{m}_F \text{LHV}} = \frac{1}{\eta \text{LHV}}$$

- The specific heat consumption shc (in J/kWh ), on the other hand, is

$$\text{shc} = \text{sfc} \times \text{LHV} = \frac{\dot{m}_F \text{LHV}}{\dot{W}} = \frac{\dot{m}_F \text{LHV}}{\eta \dot{m}_F \text{LHV}} = \frac{1}{\eta}$$

As an example, we consider a spark ICE (internal combustion engine) with a specific fuel consumption  $\text{sfc} = 245 \text{ g/kWh}$  ( = 68 g/MJ). Assigning a value of  $\text{LHV} = 44 \text{ MJ/kg}$ , we can determine the efficiency.

<sup>10</sup>The work  $T_0 \dot{S}_{G,j}$  irredeemably lost, relative to the  $j$ th transformation, can also be calculated as the change in exergy between the start and finishing states of the process. (For the exergy calculation, see Appendix B.1).

<sup>11</sup>The application of entropic analysis to various typical thermodynamic cycles is discussed in Appendix C.

The efficiency can be calculated by means of

$$\eta = \frac{1}{\text{sfc} \times \text{LHV}} = \frac{1}{68 \times 10^{-6} \times 44,000} = 0.334$$

The two parameters, sfc and shc clearly highlight how highly efficient conversion levels correspond to lower consumption of primary energy. However, other considerations need to be kept in mind, and sometimes, these prevail in the choice and planning of a plant: limiting the weight and bulk of the system, for example, or the type of fuel used. To conclude, the conversion efficiency should be the highest possible that is compatible with the production costs of the useful energy.

### 1.3 The Conversion Energy Systems and the Thermodynamic Cycles

At present, 70–80 % of the consumed primary energy derives from fossil fuels (oil, natural gas, and coal). Taking just electricity production, coal accounts for around 40 % and natural gas for 20 %. In certain countries, the coal contribution to electricity generation is far higher than the world average: USA 50 %, Australia & PR China 80 % and South Africa 90 %. In Europe, Germany produces 45 % of its electricity from coal. Meanwhile, the global coal consumption in the world is rising (+7.6 % in 2010).

The consumption of natural gas is also growing rapidly in the world (+7.4 % in 2010 compared to the year before).

The conversion of the chemical energy of fossil fuels (coal and natural gas, above all) into electricity is carried out, on a large scale, in plants of hundreds of MW using steam cycles (in the case of coal) and, predominantly, combined cycles of gas turbines and steam cycles in the case of natural gas. The natural gas is also a good source of primary energy for the distributed generation of energy, thanks to its low levels of carbon dioxide and polluting emissions (sulphur dioxide, particulates, metals) per unit of energy produced (see Table 1.1). The distributed production of electricity with medium to high power plants, possibly associated with district heating, is still usually entrusted to steam thermodynamic cycles (with water vapour or organic fluids). If the useful electrical power is modest (a few kW), it then becomes interesting, from a thermodynamic and environmental point of view, to employ external combustion engines with a Stirling cycle (where possible) alongside the internal combustion engines. The thermodynamic conversion of solar energy and the use of biomass and geothermal energy can also contribute to the production of electricity, via vapour cycles (by steam or organic fluids).

To summarise, then, at least 70–80 % of the electricity currently being produced resorts to thermodynamic engines operating in closed cycles (typically, steam cycles), associated, where necessary, with open cycle gas turbines.

The conversion of heat into electricity or mechanical energy takes place via thermodynamic cycles and fluid machines which, together with all the devices and apparatus necessary for (a) treating the fuel (e.g. in the thermo-electrical or biomass stations), (b) carrying out the combustion, (c) treating and refining the products of combustion and (d) generating electricity and distributing it, all constitute the conversion energy systems.

The engines and the turbines convert thermodynamic energy (in the form of pressure and enthalpy) or kinetic energy into mechanical energy; the pumps and the compressors convert mechanical energy into pressure heads; the heat exchangers enable the transfer of thermal power between different streams of fluids.

Apart from electricity generation, other sectors using large-scale energy conversion include (a) heat production (steam generators, cogeneration systems, heat pump systems) and (b) propulsion (propellers and turbo propellers, jet propellers).

The systems for converting energy based on the prime movers are today usually realised by means of

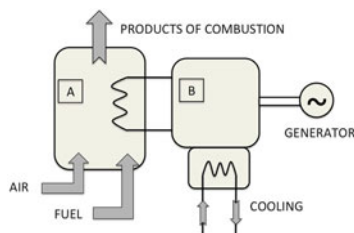
1. Rankine cycles (with water and steam). With power typically ranging from several electrical MW up to 1,000–1,800 MW. The typical efficiencies of the great thermoelectric units are  $\approx 45\%$  (with about 10 % of heat loss at the chimney stack and around 40 % at the condenser).
2. Joule–Brayton cycles (operating with a single-phase fluid). With power units ranging from a few MW to several hundreds of MW. Efficiencies are very variable ( $\approx 20\text{--}40\%$ , according to the power). For the bigger machines, the typical value is 40 %, with around 58 % of the LHV (lower heating value) lost in the exhaust gases and 2 % dissipated in other ways.
3. Internal combustion reciprocating engines. From ten or so kW to ten or so MW. Efficiency varying from 20 % to 45 % according to the power. About 35 % of LHV is lost in the exhaust gases, and about the 20 % of the inlet heat is discharged as refrigerating losses.

An important distinctive characteristic of the fluid machines used in the energy conversion systems is their speed. The turbomachines normally work at speeds of hundreds of metres per second<sup>12</sup>; the reciprocating engines have operating speeds of tens of metres per second. Since the speed of the moving parts of the fluid machines coincide with the speed of the fluids that they contain, they are directly proportional to the fluid dynamic losses.

The usual method by which available heat is transferred to the thermodynamic engine, which, operating in a closed cycle, transforms it (in part) into mechanical or electrical work, is shown in Fig. 1.8. The heat may derive from fuel oxidation (as in Fig. 1.8) or any other origin. As the engine needs to discharge a fraction of its

---

<sup>12</sup>For example, the length of the LP last-stage blade of the largest nuclear turbine is today 1.75 m, with a corresponding annular exhaust area of 25.83 m<sup>2</sup>. At a speed of 3,000 rpm, the resulting peripheral velocity is about 640 m/s. A small radial compressor (see [8]) at 250,000 rpm with a maximum diameter of 37 cm has a peripheral maximum velocity of about 480 m/s.



**Fig. 1.8** A schematic representation of an energy conversion system with external combustion. The chemical reaction between the fuel and the oxygen of the air in the combustion chamber A releases thermal energy which, transferred to the working fluid of the thermodynamic engine B, permits the generation of electricity

thermal power into the environment, it must be cooled. In the case of combustion, the gases produced by the reaction are sent to the chimney and any solid residue (the ash) is collected and disposed of.

The Rankine cycles are always with external combustion; the Joule–Brayton cycles are cycles that usually employ internal combustion. In certain niche applications, Joule–Brayton cycles have also been designed in closed circuits with thermal power from external combustion or from nuclear reactions, solar thermal energy, etc. (see Sect. 1.7).

Cogenerative systems, as particular forms of energy conversion apparatuses are briefly described in the next section.

## 1.4 The Cogeneration of Thermal and Electrical Power

Cogeneration plants use the usual thermodynamic engines and are designed in such a way as to produce electricity and heat according to the two traditional schemes shown in Fig. 1.9.

A broader and more general definition for the cogeneration processes could be the following [10]:

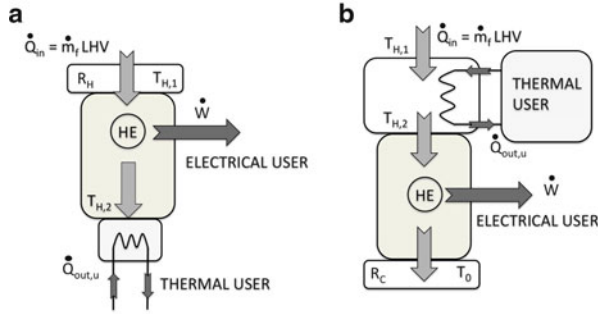
A cogeneration process is defined as one of various operations intended for the combined production of mechanical/electrical energy and heat, both considered useful effects, starting from any source of energy.

The cogeneration process needs to make a more rational use of the primary energy than the processes that produce the two forms of energy separately. The production of mechanical/electrical energy and heat should take place mainly in interconnected sequential processes.

A cogeneration system is formed by the totality of all the elements suitably combined to accomplish the cogeneration process.

The process of combined generation of electricity and heat must be “more rational” than their production separately by boiler and power station. That is, the cogeneration process must involve primary energy savings. The rational conversion





**Fig. 1.9** Scheme of cogenerative systems. **(a)** With a topping engine. The engine receives heat at high temperature and the thermal utiliser collects the heat discharged by the engine; **(b)** with a bottoming engine. The thermal utiliser employs a fraction of the heat available at high temperature and the engine recuperates the residual heat, producing electricity

of primary heat energy into mechanical/electric work and useful thermal energy is obtained by cascade combination of the various machines (engines and boilers) that produce the conversion (see Fig. 1.9).

The presence of two useful effects in the cogeneration system does not give a univocal definition of its efficiency, and it is usually a common practice to define various indices of merit. For example,

$$\text{electric efficiency} \quad \eta'_{\text{el}} = \frac{\dot{W}}{\dot{Q}_{\text{in}}}$$

$$\text{thermal efficiency} \quad \eta'_{\text{th}} = \frac{\dot{Q}_{\text{out,u}}}{\dot{Q}_{\text{in}}}$$

$$\text{energy efficiency} \quad \eta' = \frac{\dot{W} + \dot{Q}_{\text{out,u}}}{\dot{Q}_{\text{in}}}$$

$$\text{efficiency of electric energy production} \quad \eta'_{\text{el,p}} = \frac{\dot{W}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out,u}}/\eta_{\text{th}}}$$

$$\text{electric index} \quad I_{\text{el}} = \frac{\dot{W}}{\dot{Q}_{\text{out,u}}}$$

$$\text{primary energy saving index} \quad \text{ESI} = \frac{\dot{Q}_{\text{in}}^* - \dot{Q}_{\text{in}}}{\dot{Q}_{\text{in}}^*}$$

In the expression of the coefficient ESI, the term  $\dot{Q}_{\text{in}}^*$  represents the primary thermal power that would be consumed producing the heat and the electricity separately in traditional plants ( $\dot{Q}_{\text{in}}^* = \dot{W}/\eta_{\text{el}} + \dot{Q}_{\text{out,u}}/\eta_{\text{th}}$ ), whilst the term  $\dot{Q}_{\text{in}}$  represents the primary thermal power (from fossil fuel) consumed in the

**Table 1.2** Data and reference performances of the waste incinerator of Brescia

|                                               |       |
|-----------------------------------------------|-------|
| Waste mass flow, t/h                          | 66    |
| Waste LHV, kcal/kg                            | 2,326 |
| Thermal power to the steam cycle, MW          | 164   |
| Net electrical power, MW                      | 44    |
| Net thermal power to the district heating, MW | 110   |

cogeneration system to produce similar useful effects  $\dot{W}$  and  $\dot{Q}_{\text{out,u}}$ . The efficiency  $\eta_{\text{th}}$ , in the efficiency expression for the production of electricity  $\eta'_{\text{el,p}}$ , is the efficiency of a boiler that would produce a useful thermal power equal to  $\dot{Q}_{\text{out,u}}$ . The adopted values of the efficiencies  $\eta_{\text{th}}$  and  $\eta_{\text{el}}$  for the power and thermal plants dedicated to the separate production of electricity and heat are usually subject to legal regulation and calculated as appropriate averages on plants with existing technology.

## Exercises

**1.4.** The waste-to-energy facility cogeneration plant in Brescia, which started up in 2,000, exploits the combustion of solid urban waste to generate electricity and heat. It forms part of the city's district heating and by tapping the steam from the turbine, the water for the district heating can be heated. Typical operating data for cogeneration, for a winter's day and under nominal conditions, are shown in Table 1.2 [11]:

Let's evaluate the performance indices.

Assuming  $\eta_{\text{th}} = 0.90$ , applying the previously defined ratios and using the data in Table 1.2, we get

$$\begin{aligned}\eta'_{\text{el}} &= \frac{44}{66 \times 2,326 \times 4.184 \times \frac{1}{3,600}} = \frac{44}{178.42} = 0.25 \\ \eta'_{\text{th}} &= \frac{110}{178.42} = 0.62 \\ \eta' &= \frac{44 + 110}{178.42} = 0.86 \\ \eta'_{\text{el,p}} &= \frac{44}{178.42 - \frac{110}{0.90}} = 0.78 \\ I_{\text{el}} &= \frac{44}{110} = 0.40\end{aligned}$$

As far as the primary energy saving index is concerned,

$$\text{ESI} = 1 - \frac{\dot{Q}_{\text{in}}}{\dot{Q}_{\text{in}}^*} = 1 - \frac{\dot{Q}_{\text{in}}}{\dot{W}/\eta_{\text{el}} + \dot{Q}_{\text{out,u}}/\eta_{\text{th}}}$$

with  $\eta_{el}$  representing the typical production efficiency (including distribution) of the electricity. Let's assume two values for it:  $\eta_{el} = 0.42$  typical for coal power stations and  $\eta_{el} = 0.53$  referring to natural gas stations in combined cycle. Then,

$$ESI = 1 - \frac{178.42}{\frac{44}{0.42} + \frac{110}{0.90}} = 0.214 \quad \text{per} \quad \eta_{el} = 0.42$$

or

$$ESI = 1 - \frac{178.42}{\frac{44}{0.53} + \frac{110}{0.90}} = 0.131 \quad \text{per} \quad \eta_{el} = 0.53$$

The plant, considered in its winter set-up, always gives notable savings in primary energy (from 10 % to 20 %, according to the reference scenario) and favours the production of thermal energy:  $I_{el} = 0.4$ ,  $\eta'_{el,p} = 0.78$ .

The simplified evaluations carried out here refer to powers in a single working condition; more detailed energy analysis should be referred to energy produced throughout the year.

**1.5.** The TOTEM, acronym for “Total Energy Module”, was the first example of a small engine for cogeneration created in Italy, during the 1970s. It was designed by the Fiat Research Center and developed in Fiat Auto. The device used a 903 cm<sup>3</sup> engine, derived from a widely used car engine and specially adapted to work with natural gas, LPG or biogas [12, 13].

Under nominal conditions, the engine operated a 15 kW alternator that supplied electricity and made available 39 kW of thermal power, heating a capacity of 3,000 L/h of water. Under nominal conditions, fuel consumption (natural gas, LHV = 35.5 MJ/Nm<sup>3</sup>) amounted to 5.7 Nm<sup>3</sup>/h.

We can calculate the performance indices, assuming  $\eta_{th} = 0.9$  and  $\eta_{el} = 0.53$  and with the powers at nominal conditions:

$$\begin{aligned} \eta'_{el} &= \frac{15}{\frac{5.7}{3,600} \times \frac{35.5}{1,000} \times 10^6} = \frac{15}{56.208} = 0.27 \\ \eta'_{th} &= \frac{39}{56.208} = 0.69 \\ \eta' &= \frac{11 + 39}{56.208} = 0.96 \\ \eta'_{el,p} &= \frac{15}{56.208 - \frac{39}{0.90}} = 1.16 \\ I_{el} &= \frac{15}{39} = 0.38 \\ ESI &= 1 - \frac{56.208}{\frac{15}{0.53} + \frac{39}{0.90}} = 0.215 \end{aligned}$$

As shown by the results, the TOTEM system is also particularly suitable for heat production (low electrical index, modest  $\eta'_{el}$ , high  $\eta'_{th}$ , and  $\eta'_{el,p}$ , even greater than the unit).

In any case, it is highly unlikely that the analysis of a single parameter will allow the univocal thermodynamic evaluation of a cogeneration system. The effectiveness of the system should be evaluated by more complex procedures and energy balances, to be compared with other possible options for the plant, also bearing in mind the power levels. Another parameter for evaluating the thermodynamic quality of the process could be, for example, the second-law efficiency, which compares the useful powers  $\dot{W}$  and  $\dot{Q}_{out,u}$  with the maximum useful powers obtainable (electrical and thermal) for equal consumption of primary energy. Furthermore, plants generally operate with variable loads, often in ways that are difficult to predict. The various engines can use fuel of varying quality and origin, and the overall assessment must also include fundamental plant characteristics: demand for labour, reliability, maintenance, interaction with the environment (emissions, noise). Last but not least, the economic aspects (plant costs, installation, and operation) play a vital role.

**1.6.** If we connect a TOTEM engine, like that discussed in the previous exercise, directly to the compressor of a heat pump, the thermal user will receive heat from both the engine and the heat pump [14].

If the thermal power  $\dot{Q}_{in}$  consumed in the engine is equal to 100, then 27 units are available as mechanical power which, multiplied by the COP of the heat pump, supply the thermal user with  $27 \times \text{COP}$  units of thermal power, to be added to that directly recovered from the engine (69 units). Consequently, the system produces thermal energy with the relationship  $\dot{Q}_{out,u}/\dot{Q}_{in} = 0.27 \times \text{COP} + 0.69$ . From the energy point of view, the system is better than a traditional boiler if  $\text{COP} > (\eta_{th} - 0.69)/0.27$  or, assuming  $\eta_{th} = 0.9$ , for values of  $\text{COP} > 0.78$ .

Figure 1.10, obtained from data reported in [16], compares various heat production technologies: direct combustion boiler, electric heat pump, several typical systems of cogeneration and district heating plants. The comparison between the various plant types is made via the mechanical equivalent of the resource consumed  $\epsilon = \dot{W}/\dot{Q}_{out,u}$  per unit of heat produced as its temperature varies.<sup>13</sup>

The curve “a” represents the theoretical mechanical equivalent of the heat  $\epsilon$  as its temperature varies:

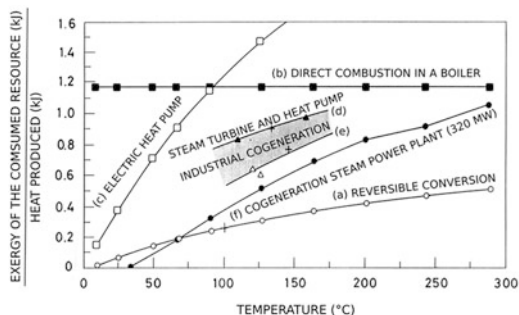
$$\epsilon = \frac{1}{\text{COP}_{\text{HP,max}}} = 1 - \frac{T_0}{T_H}$$

with  $T_C = 273 \text{ K}$  and  $T_H$  variable between 0 and  $300^\circ\text{C}$ .

Curve “a” can be associated to the straight line “b”, which gives the real energy cost of heat production in the case of a boiler ( $\epsilon = 1/\eta_{th}$ , with  $\eta_{th} = 0.85$ ). In the other cases considered, the coefficient  $\epsilon$  was calculated as the ratio between the

<sup>13</sup> That is, the heat exergy once the temperature at which the heat is made available has been fixed. For the definition and the use of exergy balances, see Appendix B.

**Fig. 1.10** Exergy cost for heat as a function of its temperature, with reference to different production technologies



electricity obtainable (but not produced) in the plant from the heat available at the desired temperature.

The great distance separating curves “a” and “b” represents the recovery margin on which innovative heat generation technologies can impact. The performances of some of these technologies are represented by curves “c” (electric heat pump), “d” (cogeneration by means of small steam plants), “e” (cogeneration by means of great industrial plants), and “f” (cogeneration by means of great electricity production plants). All these curves give the relationship between the mechanical equivalent of the resource consumed and the quantity of heat produced. Furthermore, in a scenario that would see the dotted section (obtained by extrapolating data relative to average cogenerative technologies) accessible to most end users, the concrete value of the energy resource for direct heat uses would be drastically reduced when compared to traditional boilers.

## 1.5 The Traditional External Combustion Prime Movers

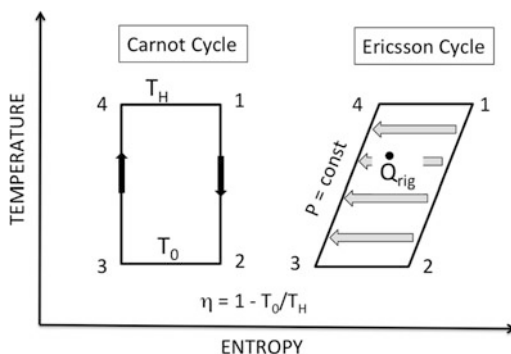
The current prime movers are the result of various restrictions and necessities, some thermodynamic, others mechanical or technological. As we have seen, the energy conversion system which is still most widespread is that which transforms thermal energy into mechanical by means of fluid machines,<sup>14</sup> so the working fluid is an important intermediary in the transformation.

The thermodynamic heat conversion comes about via an engine, inside which the fluid operates in a cycle. If the operating fluid is neither taken nor released to the surrounding environment, but always remains confined to the machine, then it is called a closed-cycle engine.

At present, the heat available is mostly produced by combustion of fossil fuels or biomass or originates from nuclear fission.

<sup>14</sup>An exception being the fuel cells, which convert the chemical energy of the fuels, usually hydrogen or methane, directly into electricity.

**Fig. 1.11** Archetypal thermodynamic cycles. Both cycles have the same performance, since they work between the same temperatures and are ideal. In the Carnot cycle, the turbomachines are wholly entrusted with raising the fluid temperature; in the Ericsson cycle, this is the task of the regenerative exchanger



As far as the machines are concerned, the turbomachines typically have small surfaces, and the fluid passes through at high speed (hundreds of metres per second. See Footnote 12). The turbomachines are, therefore, perfectly adapted to adiabatic processes (isentropically, under the ideal conditions). The heat exchangers, with continuous flows, are appropriate for isobaric processes.

Consequently, the working fluids in the real thermodynamic cycles mainly evolve according to isobar transformations and isentropic processes. An exception, albeit an important one, are:

- The isothermal transformations in the Stirling and Ericsson engines.
- The isochoric transformations in the reciprocating internal combustion engines and the Stirling cycles (the latter being external combustion).

Then, there are constraints to the minimum temperature (typically the ambient temperature) and to the maximum temperature (the temperature of the heat sources or the maximum admissible temperature imposed by technological limits). The thermodynamic cycles, it is known, see Sect. 1.1, have a good thermodynamic quality if the available thermal power  $\dot{Q}_{in}$  and the thermal power discharged into the environment  $\dot{Q}_{out}$  are exchanged at extreme temperatures (e.g.  $T_H$  and  $T_0$ ). In that way, the isothermal processes are favoured. Furthermore, the temperature difference  $T_H - T_0$  should be (for the engines) the greatest possible, and to promote this temperature difference, the methods are:

- Using turbomachines (as in the case of steam expansion in the Rankine cycles).
- Using a regenerator (as in the ideal regenerative cycles like Joule–Brayton or the Ericsson cycles or Stirling cycles).

Figure 1.11 shows the thermodynamic temperature–entropy plane with the two ideal thermodynamic cycles that can be considered as reference models for all real cycles: the Carnot and the Ericsson cycles.<sup>15</sup> In general, the Carnot cycle is taken as reference for the Rankine cycles. From the point of view of thermodynamic

<sup>15</sup>John Ericsson (1803–1889) was a Swedish-American inventor and a mechanical engineer. Ericsson invented a regenerative engine in the 1820s which uses hot air. A similar engine had been

behaviour, the (ideal) Joule–Brayton regenerative gas cycle tends towards the Ericsson cycle when the compression ratio is very small.

In general, the gas cycles use air and are open cycles. If the cycle is open, the combustion can be internal and the gases produced by the combustion act directly as working fluid in the thermodynamic cycle. Closed cycles, however, have certain advantages over open ones:

- The working fluid can be specially chosen. For example, steam (water vapour) is thermodynamically better than air if the cycle is operating at relatively low temperatures.
- It is possible to employ evaporation and, especially, isothermal condensation. As these processes are isothermal, they are thermodynamically preferable.
- The phase change implies a small work of compression. As a result, even at relatively modest temperatures and with relatively inefficient machines, it is possible to obtain useful work, with good overall thermodynamic efficiency.<sup>16</sup>
- According to the power levels required, not only the working fluid but also the operating pressures can be purposely selected.
- The external combustion generally guarantees a greater control<sup>17</sup> and freedom of choice for the fuel.<sup>18</sup> Heat can even be used directly.

Table 1.3 lists the principal thermodynamic movers used in energy conversion, with the dates of the first working prototype.

The following sections examine the traditional thermodynamic engines being used today in closed cycles: the steam cycle (Rankine cycle), the gas cycle (Joule–Brayton), and the Stirling cycle.

---

patented in 1816 by the Reverend Robert Stirling, whose technical priority of invention provides the usual term “Stirling Engine” for the device.

<sup>16</sup>In gas cycles (Joule–Brayton, but also, e.g. Otto and Diesel); the differences between the compression and the expansion works are just a consequence of the average temperature of the working fluid during the two transformations (see Sect. 1.7). As a result, in the Joule–Brayton cycles, the compression and expansion operations only differ by a factor of 2–3, with the risk that the cycle will perform very badly. In the worst cases, the difference in the two works may be zero, unless the turbomachines are really very efficient. In the reciprocating internal combustion engines, since it is relatively easy to cool the engine and by virtue of its periodic operation, it is possible to reach very high maximum temperatures. So it is always possible to obtain a significant output.

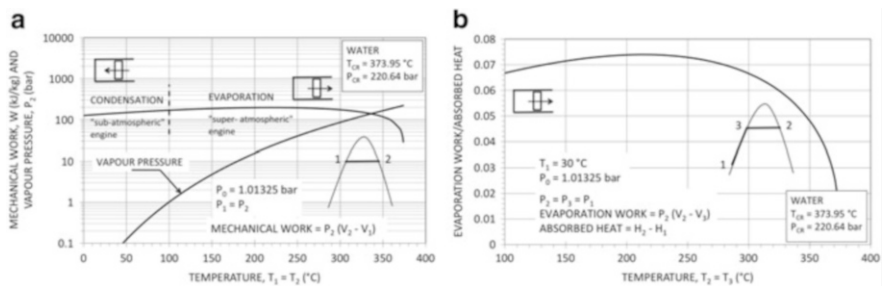
<sup>17</sup>Inside the cylinders of a reciprocating engine, for example, the residential time interval of the gas is very short and, often, the combustion is not complete. The reciprocating engines, generally speaking, are more polluting than the external combustion ones.

<sup>18</sup>Gas turbines, for example, are internal combustion engines requiring combustion gases not chemically and physically aggressive to the turbine and to the combustion chamber. In other words, the gases must not be corrosive and erosive, and they should not soil the surfaces or clog up the circuits beyond well-defined and accepted limits. These constraints place restrictions on the choice of fuels.

**Table 1.3** Thermodynamic energy converters and date of first working device

|                            |                                                  |
|----------------------------|--------------------------------------------------|
| Steam engine               | 1712 (Newcomen’s engine)<br>1769 (Watt’s engine) |
| Stirling engine            | 1816                                             |
| Otto engine <sup>a</sup>   | 1876                                             |
| Steam turbine              | 1884                                             |
| Diesel engine <sup>a</sup> | 1897 <sup>b</sup>                                |
| Gas turbine <sup>a</sup>   | 1939 <sup>c</sup>                                |

<sup>a</sup>An internal combustion engine.  
<sup>b</sup>On 17 February 1897, the official test gave the following results: full power 13.5 kW at 154 rpm, thermal efficiency 34.7 %, mechanical efficiency 75.5 % and net efficiency 26.2 % [15].  
<sup>c</sup>Year of the first experimental flight of a gas-turbine military aircraft. In the same year the first gas turbine for the generation of electricity in a public power station was installed also in Neuchatel, Switzerland.



**Fig. 1.12** (a) Water vapour pressure and work done during the vaporisation or condensation of a unit mass of water; (b) ratio between the work of expansion during the evaporation and the heat necessary for the steam evaporation

### 1.6 The Steam Cycle

The use of steam as working fluid in machines for the production of mechanical power represents the first positive attempt at industrial conversion of heat into work.

When man first seriously began to consider the possibility of substituting animal labour with the mechanical work of machines, it was discovered that most of the energy (in heat form) used for the water vaporisation was recovered as mechanical energy by great volumes of pressurised steam.

A kilogramme of water that evaporates, for example, at  $200^\circ\text{C}$ , changes phase at a pressure of 15.5 bar and produces a mechanical work equal to 196 kJ/kg (see Fig. 1.12a), and the fraction of heat converted into mechanical work (see Fig. 1.12b) is around 0.07. As the evaporation temperature rises, the pressure grows rapidly until a point beyond which the mechanical work produced and the respective conversion efficiency are diminishing. This is due to the fact that the difference between the specific saturated volumes of the liquid and the steam decreases with the evaporation



pressure. The condensation at subatmospheric pressures, with temperatures inferior to 100 °C, also makes very high work rates possible, for example, about 150 kJ/k with the steam condensing at 50 °C (see Fig. 1.12a).

The use of this mechanical energy by means of an engine was relatively easy, even though its conversion efficiency was very modest to start with.

Only subsequently did it become clear that the conversion efficiency could be significantly increased by making sure that the steam expansion happened in a determined way (according to an isentropic line). This led to the concept itself of a thermodynamic cycle: the Rankine cycle, which is still one of the most efficient thermodynamic engines for the generation of electricity.

### 1.6.1 The Thermodynamic of the Rankine cycle

The Rankine cycle is realised on the limit curve of water, and the simplest way of organising the machines (pumps, turbines, and heat exchangers) capable of creating a steam cycle is represented in Fig. 1.13a. The corresponding thermodynamic transformations for an internally reversible cycle are reported in the T-S plane (temperature–entropy) and H-S plane (enthalpy–entropy) in Fig. 1.13b, c.

Firstly, the water is taken from the minimum cycle pressure (the condensation pressure,  $P_1 = P_C$ ) to the maximum pressure  $P_2$ , by means of one or more pumps.

Then, it is subject to heating at a constant pressure (for simplicity's sake, ignoring the pressure drops) up to the evaporation temperature  $T_E = T_3 = T_4$ . This temperature is that of saturation at pressure  $P_2 = P_E$ . Then, it vaporises between points 3 and 4; the saturated vapour thus obtained is superheated up to the maximum temperature<sup>19</sup> of the cycle  $T_5$ . There follows the expansion in turbine, according to the reversible adiabatic 5–6 and condensation at temperature  $T_6 = T_C$ .

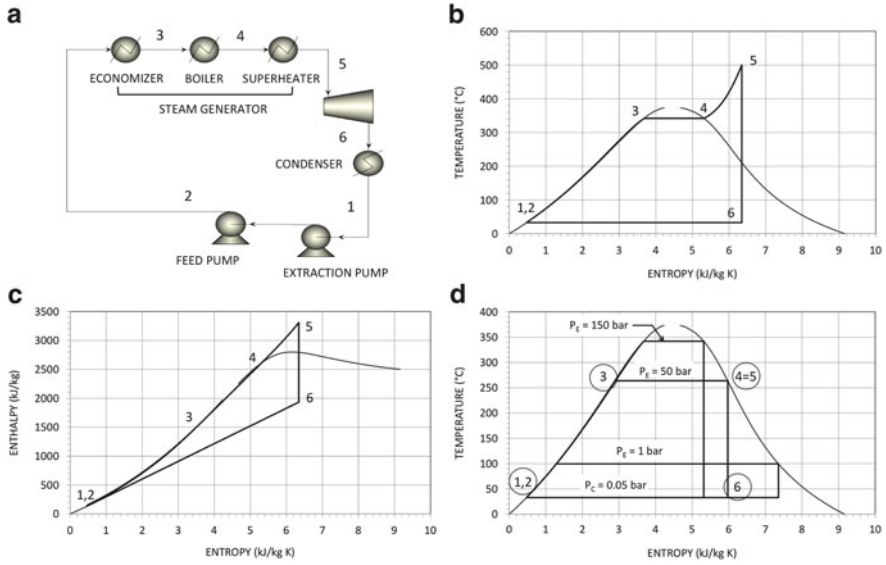
In this way, the working fluid is returned to its initial condition, having supplied its useful effect during the expansion phase in the turbine.

Generally speaking, the following considerations apply, even for the more complex cycles:

- The condensation pressure  $P_C$  is unequivocally linked to the temperature  $T_C$  (see Fig. 1.12a), which, in turn, depends on the characteristics of the refrigerating means (cold water or air) and those of the heat exchanger (the condenser).
- The evaporation pressure  $P_E$ , unequivocally linked to the temperature  $T_E$ , also represents the maximum pressure of the cycle, the choice of which will depend on economic and technical considerations.
- The maximum temperature of cycle  $T_5$  is chosen with regard to the maximum pressure (avoiding excessive condensation in the turbine) and on the basis of economic and technical considerations.

---

<sup>19</sup>If the cycle has no superheating phase, it is called saturated steam cycle. In this case  $T_5 = T_4 = T_E$ .



**Fig. 1.13** Internally reversible Rankine cycles. (a) Simplified diagram of a steam plant; (b) thermodynamic cycle in the temperature–entropy plane; (c) thermodynamic cycle in the enthalpy–entropy plane; (d) comparison between several saturated cycles at different evaporation pressures

- The turbine and the pump make transformations which can quite reasonably be considered irreversible adiabatic.

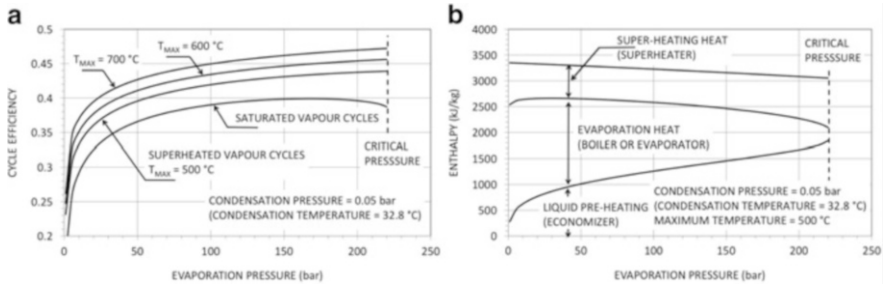
As far as the pump compression is concerned, we observe that during an ideal adiabatic compression (isentropic), a fluid generally undergoes a temperature increase that can be evaluated via the integration of the relationship

$$\left(\frac{\partial T}{\partial P}\right)_s = (-1) \frac{T}{C_p} \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial T}\right)_p$$

in which  $C_p$  is the specific heat at constant pressure and  $\rho$  is the fluid density. If the fluid can be considered incompressible ( $\rho = \text{const}$ ), the heating in a reversible compression is null and its internal energy remains unchanged.

On the other hand, the heating of the liquid due to dissipation through the pump can be calculated by hypothesising that all the hydraulic losses are concentrated at the end (or at the beginning) of the compression. In this case, the temperature increase is easily calculated as the isobar heating due to the introduction into the fluid of that percentage of work lost under the form of heat

$$\Delta T = (1 - \eta_p) \frac{W_p}{C_p} = \frac{W_p - \Delta P / \rho}{C_p}$$



**Fig. 1.14** Internally reversible Rankine cycles. (a) Thermodynamic cycle efficiency as a function of the evaporation pressure for saturated cycles and for superheated cycles; (b) heat introduced into the steam cycles at varying evaporation pressure

with  $\eta_p$  representing the hydraulic efficiency of the pump and  $W_p$  the compression work, which can be calculated (see Appendix A.2) as

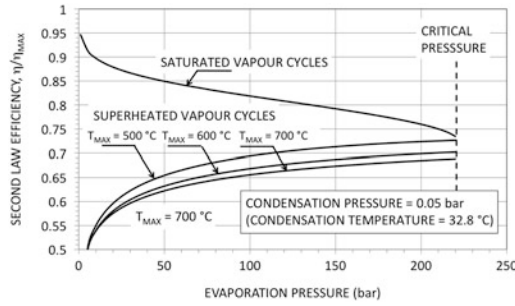
$$W_p = \frac{\Delta P}{\rho} \frac{1}{\eta_p}$$

However, the hypothesis of incompressibility is not always acceptable (see Sect. 2.1). The close link with the pump's real operating conditions means that it must be verified each time.

Figure 1.14a reports the efficiency of internally reversible Rankine cycles, with varying evaporation pressure  $P_E$ . As the  $P_E$  increases, the efficiency increases, too, because the average temperature at which the heat is introduced gets ever higher. As the  $P_E$  grows, though, the percentage of the evaporation heat diminishes (see Fig. 1.14b), because of the shape of the limit curve (see Fig. 1.13d) and consequently, especially in the case of the saturated cycles, the improvement in the efficiency is increasingly small. The effect of the maximum temperature  $T_5$  on the cycle efficiency is relatively modest. As  $T_5$  rises, though, the liquid present in the steam at the end of expansion falls, greatly improving the feasibility of the cycle and ensuring high levels of expansion efficiency.

Reducing the heat evaporation percentage and increasing the evaporation pressure reduce the thermodynamic quality (the second-law efficiency) of the cycle, and the overheated cycles have a significantly lower second-law efficiency than the saturated cycles. Figure 1.15 represents the relationship  $\eta/\eta_{max}$  for saturated cycles and superheated cycles.

At the pressure of 150 bar, for example, the internally reversible saturated cycle has an efficiency of about 0.4 and a second-law efficiency of 0.80. When  $T_5$  is  $500^\circ\text{C}$ , the first-law efficiency rises to 0.43, but that of the second law becomes 0.71. With  $T_5$  equal to  $600^\circ\text{C}$ ,  $\eta = 0.45$  and  $\eta_{II} = 0.69$ . With  $T_5$  equal to  $700^\circ\text{C}$ ,  $\eta = 0.46$  and  $\eta_{II} = 0.67$ .



**Fig. 1.15** Second-law efficiency as a function of the evaporation pressure for saturated and superheated internally reversible Rankine cycles

The relatively modest thermodynamic quality of the superheated limit cycles and the saturated cycles at high evaporation pressure is due to the irreversibilities  $\dot{S}_{G,j} / \dot{Q}_{in}$  present in the various phases of heat transmission between the heat source at temperature  $T_5$  and the working fluid in the cycle. With reference to Fig. 1.13,

$$\frac{\dot{S}_{G,2-3}}{\dot{Q}_{in}} = \frac{S_3 - S_2}{H_5 - H_2} - \frac{H_3 - H_2}{(H_5 - H_2) T_5} \quad \text{heat transfer irreversibility in the economiser}$$

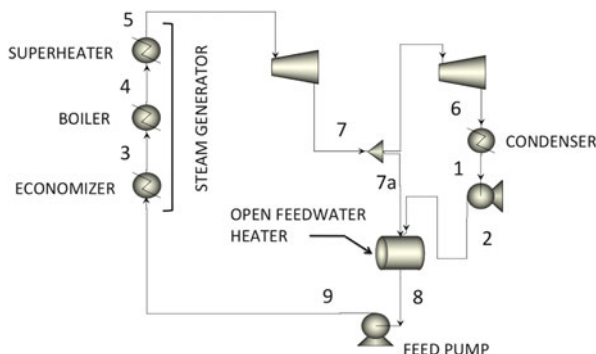
$$\frac{\dot{S}_{G,3-4}}{\dot{Q}_{in}} = \frac{S_4 - S_3}{H_5 - H_2} - \frac{H_4 - H_3}{(H_5 - H_2) T_5} \quad \text{heat transfer irreversibility in the boiler}$$

$$\frac{\dot{S}_{G,4-5}}{\dot{Q}_{in}} = \frac{S_5 - S_4}{H_5 - H_2} - \frac{H_5 - H_4}{(H_5 - H_2) T_5} \quad \text{heat transfer irreversibility in the superheater}$$

For example, at pressure  $P_E = 150$  bar and  $T_5 = 600$  °C, the irreversibility linked to the preheating of the working fluid during phases 2–3 contributes to total thermodynamic losses for 68 %, the evaporation phases 3–4 contributes 21 % and the superheating 4–5 the remaining 11 %.

On the downside, the high value of  $\eta_{II} = \eta / \eta_{max}$ , in the case of saturated cycles at low temperature (see Fig. 1.15), is the consequence of the high ratio between the heat of evaporation, introduced at a constant temperature equal to that of the hot reservoir, and the heat of the preheating.

Having established the maximum and minimum temperatures, then the increase in the thermodynamic efficiency of the Rankine cycle must pass through a reduction of the irreversibility (external to the cycle) of the heat exchange during the heating phase of the liquid, along the isobars 2–3.



**Fig. 1.16** Simplified scheme of steam cycle with one vapour extraction for regeneration

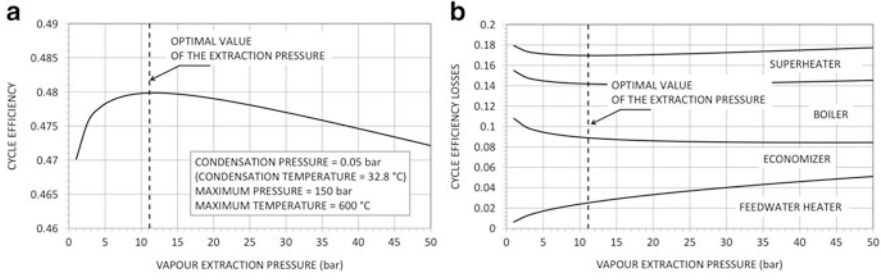
### 1.6.2 The Regenerative Rankine Cycle

As we have seen, heating the liquid in a simple thermodynamic steam cycle, as shown in Fig. 1.13, leads to a significant loss in efficiency, as it takes place under great temperature differences in the boiler. In particular, the preheating in the liquid section, from point 2 to point 3, which happens in the presence of the greatest temperature differences between the fluid and the heat source, is responsible for the largest thermodynamic loss. A substantial reduction in this heat exchange loss can be obtained by preheating, by using percentages of steam that are gradually extracted at ever lower temperatures as the vapour expands in the turbine.

Figure 1.16 shows a cycle with the steam extracted just once, at a pressure  $P_7$ , midway between the maximum pressure (the evaporation pressure  $P_E = P_5$ ) and the minimum pressure (the condensation pressure  $P_C = P_6$ ). The percentage of steam that is extracted is mixed in the open feedwater heater with the part of liquid that comes from the extraction pump of the condensate. The liquid originating from the condenser is, thereby, heated from temperature  $T_2$  to temperature  $T_8$  at the expense of the latent heat of condensation of that percentage of steam extracted in 7. The two flows, once they are mixed (at the thermodynamic conditions of point 8), proceed towards the steam generator by means of the feed pump.

Figure 1.17a reports the efficiency of the thermodynamic cycle of Fig. 1.16 as a function of the steam extraction pressure. At the optimum value of the pressure  $P_7$ , the heat exchange losses (the only ones present in the internally reversible Rankine cycle considered here) are minimal: this optimal pressure value of  $P_7$  corresponds with the maximum efficiency (see Fig. 1.17b).

In the simplified ideal case under consideration, the thermodynamic efficiency of the cycle passes from a value of 0.45 to 0.48: a net gain of three points. The open feedwater heating introduces a mixing loss, which is absent when there is no steam extraction. However, the significant reduction in irreversibility in the next phase



**Fig. 1.17** Internally reversible Rankine cycles. (a) Cycle efficiency at varying pressure of extracting the steam; (b) loss of cycle efficiency due to the irreversibility of plant components

of heating the liquid amply compensates for the minor irreversibility linked to the mixing.

The drops in efficiency  $\Delta\eta_j = T_0 \dot{S}_{G,j} / \dot{Q}_{in}$ , reported in Fig. 1.17b, with  $j = 1, 2, 3, 4$  and  $T_0 = T_1$ , can be calculated using the following relationships:

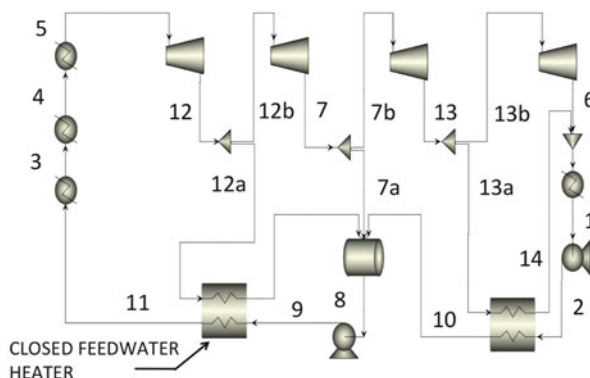
$$\begin{aligned} \frac{\dot{S}_{G,1}}{\dot{Q}_{in}} &= \frac{\dot{m}_8 S_8 - \dot{m}_2 S_2 - \dot{m}_{7a} S_7}{\dot{m}_8 (H_5 - H_9)} && \text{mixing irreversibility in the feedwater heater} \\ \frac{\dot{S}_{G,2}}{\dot{Q}_{in}} &= \frac{S_3 - S_9}{H_5 - H_9} - \frac{H_3 - H_9}{(H_5 - H_9) T_5} && \text{heat transfer irreversibility in the economiser} \\ \frac{\dot{S}_{G,3}}{\dot{Q}_{in}} &= \frac{S_4 - S_3}{H_5 - H_9} - \frac{H_4 - H_3}{(H_5 - H_9) T_5} && \text{heat transfer irreversibility in the boiler} \\ \frac{\dot{S}_{G,4}}{\dot{Q}_{in}} &= \frac{S_5 - S_4}{H_5 - H_9} - \frac{H_5 - H_4}{(H_5 - H_9) T_5} && \text{heat transfer irreversibility in the superheater} \end{aligned}$$

$$\text{with } \dot{m}_2 + \dot{m}_{7a} = \dot{m}_8 = \dot{m}_7.$$

The mixing irreversibility grows with the rising extraction pressure, whilst the irreversibility in the preheater falls rapidly. The thermodynamic losses in the boiler and the superheater remain practically steady with the pressure of the steam extraction. The rapid fall in the losses in the economiser, together with the increased losses through mixing, is responsible for the minimum value of  $\Delta\eta = \sum \Delta\eta_j$  corresponding, in our example, to an extraction pressure of around 11 bar.

In general, the greater the positive effect of the regeneration, the greater the maximum pressure of the cycle and, in practice, in modern USC (Ultra Super Critical) steam plants dedicated to electricity production, the steam extractions are at least ten or so.

The extracted steam is brought into thermal contact with the feedwater by means of surface exchangers or, more rarely, by mixing. Given the excellent exchange characteristics of both the condensing steam and the water, the differences in



**Fig. 1.18** Simplified scheme of a steam cycle with three steam extractions for regeneration

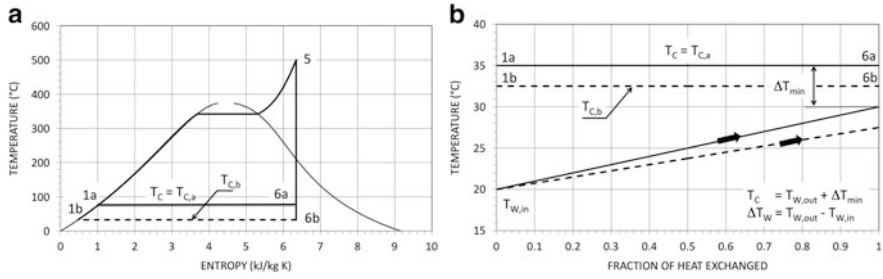
end temperature, when exiting the exchanger, are generally very small (3–5 °C). Figure 1.18 represents a cycle with two closed feedwater heaters and one intermediate open feedwater heater.

The basic criterion for establishing the route of the condensed steam at the exit of each exchanger is still that of reducing the irreversibility as much as possible or, what amounts to the same thing, the temperature difference between the different fluids in thermal contact. This explains the passage of the condensate in cascade from one exchanger to that immediately before (see Fig. 1.18).

The mixers, by consenting null differences in end temperatures, have a higher thermodynamic efficiency. However, they oblige the plant to undergo a further imposition, namely, each exchanger must be fed by a pump, which applies the same pressure of the extracted steam to the feedwater. The use of surface exchangers, though, means that the number of pumps can be drastically reduced. Commonly, at least one mixer is always present to carry out the role of a deaerating heater.<sup>20</sup>

Once the general layout of the cycle and the number of regenerators have been decided, there remains the question of determining the optimal temperature for the end of regeneration and the distribution of the feedwater enthalpy increase among the various exchangers.

<sup>20</sup>The water, which constitutes the working fluid for the power stations, must be not only demineralised but also non-aerated. In fact, at high temperatures, the oxygen from the air which has dissolved in the water becomes corrosive for iron materials. This degassing process is carried out with a special device, the deaerator, consisting of a mixture heat exchanger in which the liquid, shaped as thin sheets and free-falling, exposes to the gas-phase large surfaces for heat and mass exchange. The dissolved gas, in first approximation, even within the liquid, obeys the law of perfect gases and, in the absence of atmospheric air, expands (abandoning the liquid) and tends to assume a specific volume that corresponds to the partial residual pressure of the air in the exchanger. This pressure is kept low, driving away the air as it is gradually separated. Although, in principle, the degassing is possible at any temperature, experience teaches that it is most efficient at 100–150 °C.



**Fig. 1.19** Effects of a decrease of the condensation temperature on a Rankine steam cycle. (a) Thermodynamic cycle in the T-S plane. (b) Diagram of heat exchange at the condenser

The definite resolution of the problem is not simple as the final result depends on a great number of variables, such as the cycle layout (number of superheatings, the pressure, and temperature at which they take place), the efficiency of the rotating machines and the static equipment and the thermodynamic cycle of the gases produced by the combustion (in the case of power plants, using fossil fuels). The basic concept remains that of the overall reduction of irreversibility, as achieved in the simple case of the system in Fig. 1.16, with the results shown in Fig. 1.17.

It is not uncommon for structural and economic problems to overshadow the strictly thermodynamic aspects (e.g. the choice of end temperature for the regeneration could depend upon economic considerations, bearing in mind, for example, that the cost of the steam generator strictly depends on the degree of regeneration too). The final decision on the number of regenerators, therefore, depends on optimising the unit cost of the energy produced.

### 1.6.3 The Importance of the Condensation Pressure

The pressure at the end of expansion (the condensation pressure) has a notable influence on the thermodynamic performance and characteristics of the steam cycle.

For example, [47] referring to Fig. 1.19a, if the expansion progresses from point 6a to point 6b, that is, the condensation temperature passes from  $T_{C,a} = T_C$  to  $T_{C,b}$ , the heat introduced into the cycle grows by a quantity  $\Delta Q_{in} = (H_{1a} - H_{1b}) \approx C_P (T_{1a} - T_{1b}) = C_P (T_{C,a} - T_{C,b}) = C_P \Delta T_C$ , whilst the useful work increases by a term  $\Delta W$  equal to the area (6b - 1b - 1a - 6a):

$$\Delta W \approx (T_{C,a} - T_{C,b}) (S_{6b} - S_{1b}) \approx \Delta T_C \frac{\Delta H_{VL}}{T_C} x_{6b}$$

with  $x_{6b}$  representing the vapour quality of the point 6b.



The drop in the condensation temperature has a positive effect on the thermodynamic performance of the cycle, if the relationship

$$\frac{\Delta W}{\Delta Q_{\text{in}}} = \Delta T_{\text{C}} \frac{\Delta H_{\text{VL}}}{T_{\text{C}}} x_{6b} \frac{1}{C_{\text{p}} \Delta T_{\text{C}}} \approx x_{6a} \frac{\Delta H_{\text{VL}}}{C_{\text{p}} T_{\text{C}}}$$

assumes values greater than one unit. Assuming a reference condensation pressure equal to 0.05 bar and a vapour quality  $x_{6a}$  of about 0.9, we obtain

$$\frac{\Delta W}{\Delta Q_{\text{in}}} \approx x_{6a} \frac{\Delta H_{\text{VL}}}{C_{\text{p}} T_{\text{C}}} = 0.9 \frac{2,424 \times 10^3}{4.2 \times 10^3 \times 305.55} = 1.7$$

Therefore, the original cycle undoubtedly benefits thermodynamically from reducing the condensation temperature. The actual convenience of the procedure, or better the optimisation of the condensation temperature, is subordinate though to calculating the unit cost of the electricity produced, as a function of the condensation temperature.

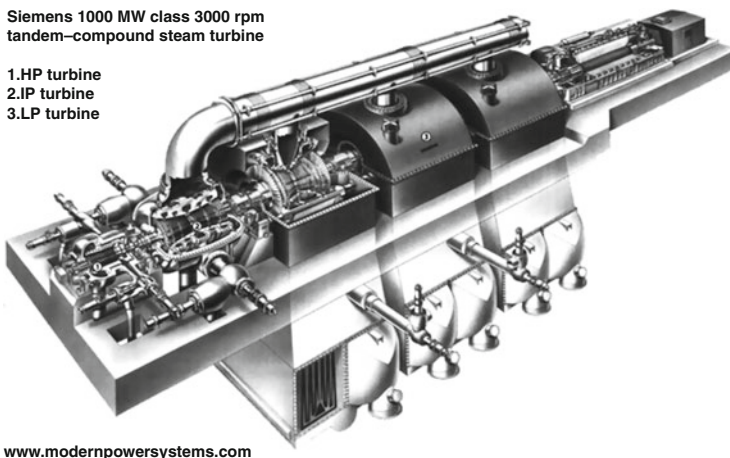
To that end, refer to Fig. 1.19b, where we report the diagram of heat exchanged in the condensation phase of a steam Rankine cycle. The temperature  $T_{\text{C}}$  is set on the basis of the temperature of the refrigerating water  $T_{\text{W},\text{in}}$ , on the basis of temperature increase  $\Delta T_{\text{W}} = T_{\text{W},\text{out}} - T_{\text{W},\text{in}}$  and on the basis of the minimal temperature difference  $\Delta T_{\text{min}}$  of the two streams in thermal contact. A lowering of the condensation temperature, for a given temperature of the refrigerating water, brings about either a drop in  $\Delta T_{\text{W}}$  (reflected in a greater flow of refrigerating water) or a drop in  $\Delta T_{\text{min}}$  (reflected in an increase of the exchange surface in the condenser) or in both of these.

With great quantities of cold water available, it is possible to push the condensation pressure down very low. For example, 0.025 bar for  $T_{\text{W},\text{in}} = 6^\circ\text{C}$ ,  $\Delta T_{\text{W}} = 8^\circ\text{C}$ ,  $\Delta T_{\text{min}} = 7^\circ\text{C}$  and  $T_{\text{C}} = 21^\circ\text{C}$ .

In conclusion, whilst the cycle certainly benefits thermodynamically from the drop in condensation temperature, the following factors have a negative impact:

- The increase in the exchange surface and, consequently, in the cost of the condenser
- The increase in the power and, therefore, in the fixed and operating costs, in the circulating pumps for the cooling water
- The increase in capacity and, thereby, the cost of the extractor for the non-condensing gases in the condenser
- The increase in the dimensions of the exhaust ring on the turbine

This last item is often the most costly. In fact, the volumetric capacity, in passing from a condensation pressure of 0.05 bar to a pressure of 0.025 bar, increases by 1.8–1.9 times. In a turbo-alternator with limited power (50–100 MW), for example, this may mean substituting the single-flow solution at low pressure with a double-flow alternative, making the machine considerably more expensive.



**Fig. 1.20** Tandem-compound steam turbine and the shell and tube condenser below the turbine. 1,000 MW class turbine at 3,000 rpm (from author [17])

Increasing the power of the pumps for the refrigerating water and of the extraction devices for the air from the condenser further impacts, in the real case, the reduction in fuel consumption that was calculated in the ideal case. Finally, reducing the exhaust pressure of the turbine increases the percentage of condensate, with fluid dynamic and technological damages (passing from a condensation pressure of 0.05–0.025 bar, the vapour quality drops, for instance, from 0.90 to 0.88). The optimal value for  $\Delta T_{\min}$  is about 5 °C, and, at least in the Italian climate, an average annual value of the condensation temperature of 32 °C is considered a realistic reference: this corresponds to a saturation pressure of 0.05 bar.

Figure 1.20 represents a modern steam turbine for a modern thermoelectric station (that of Niederaussem K, Germany) in the tandem-compound configuration, with a single-flow HP (high-pressure) section, with a double-flow IP (intermediate pressure) cylinder and with three double-flow LP (low pressure) cylinders for the condensing section, on a full speed single shaft at 3,000 rpm. The shell and tube condenser is located beside the exhaust sections of the LP section.

Since the quantity of heat that needs to be released into the environment may be greatly superior to the useful work produced (see (1.5) and Fig. 1.2), the quantity of water necessary for the condensation may be remarkable (e.g. 60 times the flow of steam to the condenser; see Problem 1.7), and such a quantity of water is not always available. Furthermore, given the great thermal power involved, there may be a significant thermal alteration in the natural water bodies, with harmful effects on the ecosystem (of a physical, chemical, and biological nature). In many countries, there are regulations governing the discharge of hot water into bodies of natural water, with the express intent of limiting excessive increases in water temperature and preventing the formation of “thermal barriers.” Consequently, alternative cooling systems are resorted to, and the system currently most in favour is that of cooling towers.



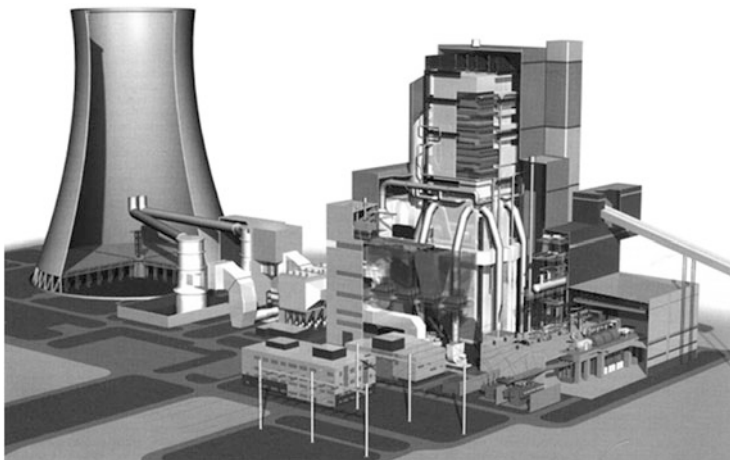
**Fig. 1.21** Calder Hall power station. Calder Hall had four natural draught cooling towers, each one 88 m high, built between 1950 and 1956 (Source Magnox Electric Ltd)

The wet cooling towers or the evaporative cooling towers, for the most part natural draught cooling towers, are usually made of great hyperbolic shells, standing vertically, with openings at both the top and the bottom and with a shallow permeable layer at the bottom end (see Figs. 1.21 and 1.22). The hot water, on leaving the condensers, is sprayed from above onto this layer, creating a wide contact surface with the air, in order to exchange mass and heat. The saturated hot column of air (which is lighter for both these reasons), which emerges from this layer, rises upwards, thanks to the chimney effect of the shell, summoning more air, which is relatively dry, from the outside through the openings at the base of the tower. The transfer of the latent heat of vaporisation through the saturation of the air cools the water, which is collected at the bottom and sent once more to the condensers. Water consumption is limited, but not unimportant (around 2 kg per electrical kWh).

Where there is no adequate natural availability, a further option is the direct exchange of heat with the atmosphere by means of devices called dry cooling towers. These consist of heat exchangers with a large surface on the air side and a tubular primary surface, inside which there flows either, directly, the vapour to be condensed (direct dry cooling condensers) or, indirectly, as in the indirect dry cooling tower, a flow of water for the purpose of cooling the condensers.

The very modest thermal capacity of air implies the use of enormous flows, whilst the small external exchange coefficients require the use of vast surfaces. The problems of the heat exchange are similar to those found in car radiators.

Whilst the evaporative towers incur a limited cost increase in the energy generated (greater investment and an increase in the condensation temperature, albeit limited), the dry towers imply significant extra costs due not only to the further



**Fig. 1.22** Carbon capture and storage (CCS) pilot plant in Niederaussem, Germany. *On the right:* lignite conveyor, turbine building and steam generator, the double gas desulphurisation system and the cooling tower. The cooling tower serves also as a chimney for the units. The cooling tower is with natural air draught (from author [18])

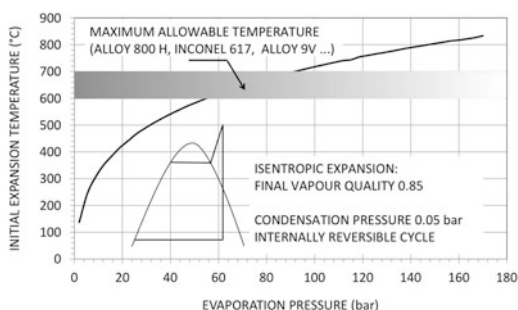


**Fig. 1.23** Detail of an air-cooled condenser of the combined power plant of Gissi (Chieti, Abruzzo, Italy). Two units of 400 MW each one. In the figure, the ducts of the steam and the fans for the cooling air are visible

investment but also to the not-insignificant rise in the condensation temperature and the energy consumption for the fans that guarantee the flow of air through the heat exchanger.

Figure 1.23 shows an air condenser, in which the exhausted steam from the turbines is cooled directly by air.

**Fig. 1.24** The inlet turbine temperature for ideal steam cycles ensuring a vapour quality at the end of the expansion equal to 0.85 as a function of the evaporation pressure

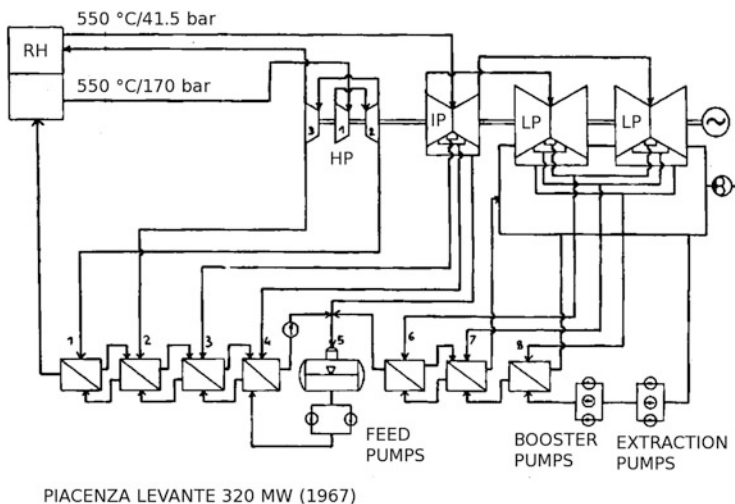


The use of cooling towers resolves partially (if wet) or completely (if dry) the problem of consumption and the excessive heating of natural bodies of water, but it does present new ones: the alteration of the local microclimate and chemical pollution (for wet towers) as well as noise pollution for the dry towers. Furthermore, the plant becomes more sensitive to changes in the environmental conditions, and during the course of the year, the discharge pressure of the turbine may vary considerably. In conclusion, the choice between conventional systems and cooling towers should be based on an overall economic evaluation that also takes into consideration the social costs due to pollution and not just the simple running costs of the power station.

#### ***1.6.4 The Superheating and the Repeated Superheating***

Almost all the steam cycles foresee superheating, the main function of which is to increase the vapour quality at the end of expansion. In fact, an excessive liquid content causes erosion of the blades in the turbine. Qualities of 0.85–0.95 are considered at the limit of acceptability. A second reason for favouring superheating is the increased internal efficiency of the turbine stages as the quality increases: on the basis of a simple empirical rule, the efficiency of a stage that works with wet steam is equal to that in the superheated region multiplied by the quality. Lastly, there is a third influence that favours superheating: the thermodynamic improvement caused by introducing added heat at high temperature.

Historically, since the middle of the last century, maximum cycle pressures have shown a significant upward tendency, pushed by technological progress and the increased efficiencies this made possible. At pressures of 120–180 bar and for start of expansion temperatures of 540–600 °C, the superheating does not guarantee a sufficient quality at the end of expansion. Figure 1.24, for example, shows, as a function of the vaporisation pressure, the superheating temperature necessary to maintain the quality at the end of an isentropic expansion equal to 0.85. As shown, the superheating temperature needed grows rapidly with the evaporation pressure.

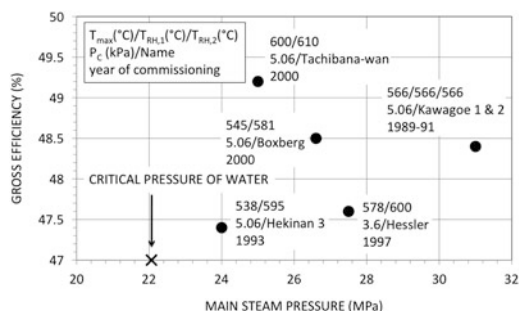


**Fig. 1.25** The simplified scheme of a real thermoelectric steam cycle. The Piacenza–Levante Power Plant (Italy), one of the two similar conventional units erected between 1965 and 1967 in Italy. Before the revamping in 2005 with a modern gas-turbine combined cycle

In principle, then, the use of higher temperatures ( $700\text{--}800\text{ }^{\circ}\text{C}$ ) could bring the ratio to an acceptable level, but such temperatures mean using sophisticated materials, especially for the steam generator where the amount of metal used is significant. Consequently, for economic reasons rather than technical ones, there has been no move to further increase the maximum temperature. As an alternative, the current practice is to resort to a second superheating, after a first expansion stage in the turbine. The entire steam flow is extracted downstream from a high-pressure group and sent to the superheater in the steam generator, from where it returns to the turbine room at maximum temperature. The benefits of resuperheating are identical to those of a simple superheating.

The double transfer of steam from the turbine to the boiler and back again, plus the added complexity required in the turbine and boiler for the resuperheating phase, makes this practice relatively expensive: nevertheless, the increased cost is amply compensated by the improved performances of the cycle. In certain cases, the standard plant scheme is replaced by a variant (typically, e.g. in steam cycles in nuclear power plants, see Sect. 1.6.6) which involves in direct superheating of the expanded steam by means of the live steam produced by the boiler; naturally, given the differences in temperature needed to enable the heat exchange, the resuperheating temperature, in this case, will be somewhat lower than that of the live steam.

Figure 1.25 shows a simplified drawing of a steam cycle (the old power station of Piacenza Levante, Italy) with superheating at  $550\text{ }^{\circ}\text{C}$ , followed by a resuperheating at the same temperature, but with an intermediate pressure of 41.5 bar. The steam



**Fig. 1.26** Performances of supercritical steam plants. The rated outputs of the plants are from 700 to 1,000 MW. In the figure,  $T_{\max}$  indicates the maximum steam temperature;  $T_{RH,1}$  and  $T_{RH,2}$  are, respectively, the superheating temperature and the resuperheating temperature (where relevant).  $P_C$  indicates the turbine back pressure (the condensation pressure)

is extracted eight times for the regeneration. Extraction number 5 concerns the deaerator. Given the high power level, the boiler feed pumps are often dragged by an auxiliary turbine, fed by steam that is tapped when required, usually at the same pressure as the deaerator.

### 1.6.5 Supercritical Cycles

The cycles that we have discussed so far have involved the isothermal vaporisation of a preheated liquid. As technology has progressed, it has become possible to create cycles with pressures greater than critical (221.3 bar) in which the isothermal vaporisation is replaced by transition to a temperature that is constantly rising, from a liquid state to a vapour one. In Fig. 1.26, from data reported in [19], we can see the gross efficiency as a function of the main steam pressure for various supercritical steam plants. Together with supercritical cycles at pressures of 250–300 bar, it has become technically and economically feasible to propose double resuperheating. However, those power stations that are capable of handling it are still in a minority. Designing the resuperheating section becomes complicated, adding a second turbine section at high temperature and, above all, bringing in additional steam-lines that have a significant diameter. For these reasons, bearing in mind the relatively modest increase in efficiency (around 1 %), those plants with maximum pressures below 300 bar (30 MPa) have preferred not to adopt a second superheating. The maximum efficiencies obtainable for temperatures lower than 600 °C stand at around 47–48 %, corresponding to a plant efficiency of 42–43 % (steam generator efficiency about 0.9). Although there have been cases of temperatures of 650 °C being used, in general, temperatures of no more than 540–560 °C are favoured because this minimises the use of steel superalloys which are not only extremely



expensive but also difficult to use. After all, the benefits that are derived from increasing the temperature are relatively modest, since the barycentre of the cycle is determined more by the shape of the limit curve than the further extension of the superheating towards higher temperatures. The European programme AD700 (“Advanced supercritical PF power plant operating at 700 °C” [20]), in existence since 1998, foresees the creation of cycles with a maximum steam pressure of 350 bar, superheating of 700 °C and resuperheating (at a pressure of 75 bar) of 720 °C. The net plant efficiency foreseen, with cooling towers, is 50–52 %.

### ***1.6.6 The Steam Cycle for the Nuclear Power Plants***

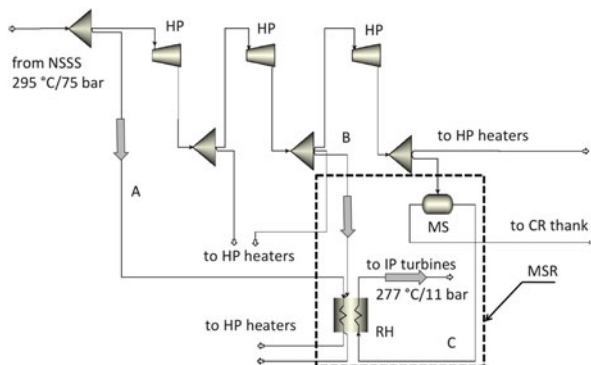
In the nuclear reactors moderated and cooled by light water, pressurised or boiling, which currently form the great majority of nuclear power plants operating today, the steam originating from the “Nuclear Steam Supply Systems” (NSSS) has a pressure of around 70–80 bar and is saturated (temperatures of 290–320 °C). The thermodynamic characteristics of the steam under these conditions give rise to a series of problems, the solution of which makes the steam cycle for nuclear power plants special.

The relatively modest enthalpy drop available (e.g. 1,000 kJ/kg, compared to 1,500 kJ/kg in the case of steam at 500–600 °C and 180 bar) and the lower thermodynamic efficiencies imply the use of correspondingly much higher flow masses (at similar horsepower), which, given the usual condensation pressures, gives rise to exceptionally high flow volumes at the discharge of turbine bodies operating at low pressure. For this reason, the nuclear turbines always have a rotating speed of 1,500 rpm (in the 50 Hz systems) or 1,800 rpm (in the 60 Hz systems), which is half the rpm usually adopted in the more traditional thermodynamic steam cycles.

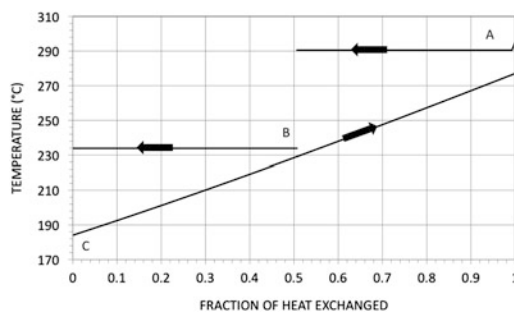
By starting expansion from near-saturation conditions, the humidity in the steam grows rapidly, creating a potentially intolerable erosion of the rotor blades in the turbine and reducing its fluid dynamics performances, too. There should always be a moisture separator for the entire steam flow, generally positioned between the bodies at high and intermediate pressure, followed by a slight intermediate superheating with live steam. Figure 1.27 represents a basic layout for drying the steam and the resuperheating in two stages in a block called MSR (Moisture Separator Reheater). This layout is that adopted by the power station of Flamanville 3 [21].

The qualitative diagram of heat exchange in the superheater RH of the block MSR is shown in Fig. 1.28. The steam with thermodynamic conditions of point C is superheated by means of two extractions A and B at two different pressures. The steam A is a portion of the live steam derived from the NSSS.





**Fig. 1.27** Basic layout of the drying system and steam superheating for a turbine of a nuclear reactor



**Fig. 1.28** Diagram of the heat exchange in the superheater RH of the MSR system from this figure

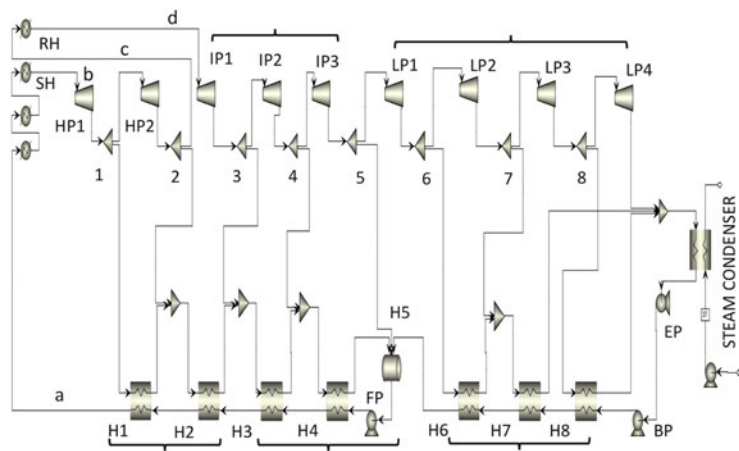
## Exercises

**1.7.** Figure 1.29 shows the plant layout (simplified) of a thermoelectric plant of 320 MW (the old power station of Piacenza–Levante, Italy; see Fig. 1.25), and Table 1.4 has the basic parameters adopted for the cycle calculations.

The regenerators H1 and H2 are fed with two steam extractions from the HP stages of the turbine. The surface regenerators H3 and H4 and the deareator H5 are fed by steam extractions from the IP stages of the turbine. The regenerators H6, H7, and H8 use steam extracted from the LP turbine.

The cycle efficiency (excluding that of the boiler) is calculated as the ratio between the useful horsepower and the thermal power entering the cycle.

$$\eta = \frac{(\sum \dot{m}_{HPi} W_{HPi} + \sum \dot{m}_{IPi} W_{IPi} + \sum \dot{m}_{LPi} W_{LPi}) - (\dot{m}_{FP} W_{FP} + \dot{m}_{BP} W_{BP} + \dot{m}_{EP} W_{EP})}{\dot{m}_a (H_b - H_a) + \dot{m}_c (H_d - H_c)}$$



**Fig. 1.29** Scheme adopted for calculating the basic thermodynamic characteristics of the cycle in Fig. 1.25

**Table 1.4** Basic assumed parameters for the calculations of the steam Rankine cycle of Fig. 1.29

|                                                                   |                                                                          |
|-------------------------------------------------------------------|--------------------------------------------------------------------------|
| Maximum temperature                                               | 550 °C                                                                   |
| Reheating temperature                                             | 550 °C                                                                   |
| Turbine inlet pressure                                            | 170 bar                                                                  |
| Reheating pressure                                                | 41.5 bar                                                                 |
| Condensation pressure                                             | 0.043 bar                                                                |
| Efficiency of the turbines                                        | 0.77 (HP1) 0.8 (HP2)<br>0.85 (HP3)<br>0.88 (for the IP and LP cylinders) |
| Efficiency of the pumps                                           | 0.85                                                                     |
| Minimum temperature difference in the closed feedwater heaters    | 5 °C                                                                     |
| Subcooling degree at the output of the open feedwater heaters     | 2 °C                                                                     |
| Subcooling degree on the hot side of the closed feedwater heaters | 5 °C                                                                     |
| Pressure drops:                                                   |                                                                          |
| All in the steam generator (for simplicity)                       | 60 bar                                                                   |

Assuming  $\dot{m}_a = 1 \text{ kg/s}$ , we obtain a turbine power equal to 1, 202.4 kW and an input from the pumps equal to 29.69 kW. The thermal power entering the cycle is 2,505.4 kW and the cycle efficiency is  $\eta = 0.468$ .

The ratio between the useful horsepower and the power necessary at the pumps  $1,172.7/29.68 = 39.5$  is very high. This is due to the modest specific work required for the compression of the liquid and the high enthalpy drop on the turbines.

The expansion ratios for the various turbine groups are as follows: 4 for the high-pressure turbines, 6.15 for the stages of intermediate pressure and 156.98 for the stages of low pressure. The total expansion ratio is equal to 3,953. These extremely high expansion ratios correspond to an equally great variation in the flow volume, mitigated in part by the steam extractions for the regeneration: 2.86 for the HP turbines, 3.98 for the IP turbines and 62.95 for the LP turbines. The global volume ratio of expansion is 878.2.

Consequently, the steam turbines are very complex machines: with numerous stages (needed to elaborate the high enthalpy drop) and with a tendency to small flows at the first stages (with possible problems of insufficient height of blades) and very high ones at the last stages (with the need to resort to multiple flows).

The flow of cooling water to the condenser is 60.8 kg/s for every kg of steam (if the water is at 20 °C, assuming a minimum temperature difference in the condenser of 5 °C and a temperature rise in the cooling water of 5 °C). If the condensation pressure passes from 0.043 bar to 0.035 bar, the flow of water necessary for the condenser increases by 2.96 times.

The ideal efficiency is  $\eta_{\max} = 1 - (20 + 273.15) / (550 + 273.15) = 0.644$ , and the second-law efficiency is  $0.468 / 0.644 = 0.727$ . Assuming a reference temperature  $T_0$  of 20 °C, it is possible to calculate the energy losses (or the losses of thermal availability; see Appendix B). Exergy losses in the phases of heating, evaporation, and superheating account for about 52 % of the total efficiency loss with respect to the ideal case. Adding in the exergy losses at the condenser, we reach 60 %. The remaining dissipation of thermodynamic availability is due to the regenerators, the turbomachines, and the pressure losses.

From the above, it should be clear how the great power steam cycles are characterised by very complex plant layouts: superheating followed by resuperheating, maximum pressures of 150–180 bar even reaching supercritical levels (up to 300 bar) and numerous regenerators for preheating the feedwater to the boiler. Such complexity is justified by the high power typically required by the great steam power plants. Since the early twentieth century, the unit power of the fossil steam turbine power plants has grown from just a few MW to the present 800–1,000 MW. The nuclear power plants have reached levels of 1,000 MW in even shorter time: the first commercial power plant, Calder Hall, in Sellafield, which started operating in 1956, had an electric power of 50 MW; the EPR reactor (a third-generation reactor, pressurised, cooled and moderated by light water) has a declared power of 1,750 MWe.

The sophisticated plant layout (aimed at reducing both internal and external irreversibilities of the cycle) and the rise in pressures and maximum temperatures have led to the current high efficiencies of cycle: from values of 15–20 % in the 1900s to the modern 45–50 % of fossil steam turbine power plants. The light-water nuclear power plants, due to their low operating temperatures, currently have efficiencies around 35–37 % (but, for this question, see exercise 1.2).

In the last few decades, the water steam cycle has seen a rapid and significant acceptance in combined cycles (see [22]), as the cycle for the recovery of thermal

energy at the discharge of modern gas turbines.<sup>21</sup> The typical steam cycles for these applications have the special characteristic of several evaporation phases (two or three), in such a way as to reduce the irreversibilities (the dissipation of the thermodynamic potential) of the heat exchange during the cooling phase of the turbine discharge gases, which constitute a heat source of variable temperature.

Technological development has made steam cycles highly versatile and meant that they are widely used in all industrial sectors. Just think, for example, of the numerous plant set-ups for steam cycles that are used in the cogeneration of electricity and heat or the power plants for the thermodynamic conversion of solar energy, the CSP (Concentrated Solar Power) plants: using a parabolic-trough collector or based on a central tower or on Fresnel reflectors. As far as the power levels are concerned, the steam cycles are practically without rivals in applications using power supplies greater than a few dozen MWe up to (as we have seen) the extremes of 1,000–1,500 MW. It is rather difficult to realise efficient steam turbines for small or modest power levels.

## 1.7 The Closed Gas-Turbine Cycle

The first patent concerning a heat engine based on the use of a gas turbine dates back to 1791 and carries the name of an Englishman, John Barber. In 1904 Franz Stolze, a German, built a prototype by using an axial compressor and a multistage reaction turbine mounted on the same shaft. The air from the compressor was preheated, prior to the combustion, using the hot expansion gases from the turbine. The turbomachinery used, though, did not possess the performance levels needed to achieve useful work (also given that the maximum temperatures permitted by the material in use in those years were, in all probability, not very high).

The gas cycle is also known as the Brayton cycle, after George Brayton (1830–1892), an American engineer who successfully constructed a gas engine in the 1870s, operated by means of a thermodynamic cycle consisting of two isobars and two isentropes. The expansion and compression were produced by volumetric machines. Only in the twentieth century, with the availability of appropriate materials (resistant at high temperatures) and following a substantial perfecting of the fluid dynamics of the turbomachinery, was it possible to make the first working gas turbines, that is, those capable of supplying useful power.

---

<sup>21</sup>The Korneuburg A power station (Austria) was, in 1961, the first true combined gas-steam cycle to enter service. The power station used two gas turbines of 25 MW each and a steam turbine of 25 MW, too. The overall efficiency was around 32 %. Modern combined cycles using natural gas as fuel can reach plant efficiencies of nearly 60 % and typical power levels of 400 MW.

The first patent for a closed gas cycle is dated 1935 and refers to the prototype of a 2,000 kW pilot plant, built in Zurich in 1939: the AK-36 Plant<sup>22</sup> in the Escher Wyss factory.

In continuous flow machines, the work is proportional to the specific volumes.<sup>23</sup> To obtain useful work in a thermodynamic cycle, it is therefore necessary for the mean specific expansion volumes to be bigger than those of the compression phase. There needs to be a dilatation mechanism for the volumes, which, in the steam cycles, is represented by the vaporisation. The gas cycles do not have such an efficient instrument, in fact, if the vaporisation multiplies volumes a hundredfold, making the expansion work far greater than the compression, the gas cycles have to rely on dilatation caused by induced heating (generally in open cycles) from combustion: a threefold increase in the mean absolute temperature of the turbomachines, for example, gives an expansion work that is ideally three times that of compression and, therefore, to a certain extent, comparable to it. The differing quantities of useful work that the steam and gas cycles provide mean a radical difference in the incidence of thermodynamic losses (see Appendix C). In the case of steam, even where the dissipations in the turbine and the pump are very high, there is no risk of the expansion work approaching that of the compression, however, in the case of the gas cycle, the combined effect of the losses in diminishing the work of the turbine and increasing that of the compressor has a radical effect on the useful work, with the risk, should the machines be only moderately efficient, of nullifying it all together. For this reason, the ability to manage the high temperatures, increasing the intrinsic work difference between the turbine and the compressor, and the quality of the fluid dynamics of the turbomachines play a fundamental role in gas-turbine cycles. In modern industrial applications requiring great power, the gas-turbine cycle is, usually, in an open cycle and with internal combustion. The gas turbine is often associated with a recuperative steam cycle. The gas-turbine cycle with internal combustion (on open cycle), although attractive for the simplicity of the plant, the lightness of the turbo-compressor, the freedom of choice it gives in plant location, thanks to its ability to function without cooling water and under a wide range of climactic conditions, has a performance which is nevertheless heavily influenced by the nature of the working fluid (combustion gases) and the basic cycle pressure (atmospheric), both of which are imposed by the external environment.

The closed gas cycle has frequently been proposed for special applications: when using solid fuels or those of low quality and cost (coal, biomass, waste, syngas), waste heat recovery, solar energy, nuclear energy,<sup>24</sup> etc.) (see [23–25]). If, on the one hand, choosing a closed cycle means introducing certain new

---

<sup>22</sup>It was named after the inventors Jakob Ackeret (1898–1981), a Swiss aeronautical engineer and a pupil of A. Stodola, and Curt Keller.

<sup>23</sup>The specific adiabatic work is equal to the difference in enthalpy:  $W = \Delta H$  (see Appendix A.2). Then,  $(\partial H / \partial P)_S = 1/\rho$ .

<sup>24</sup>In principle, both solar and nuclear sources are suitable for closed-cycle gas turbines intended for applications in space (e.g. space missions).

limitations, represented mainly by the primary heat<sup>25</sup> and waste heat exchangers and the need to have an appropriate cooling fluid, on the other hand, it affords two additional freedoms: (a) the choice of a working gas with preset characteristics and (b) that of a plant pressure level (the base pressure of the cycle) which affords the maximum economy for the turbogenerator-heat exchanger unit. In fact, the size of the turbomachines and the exchangers is heavily influenced by the density of the working fluid and, hence, by its pressure.

### 1.7.1 The Isentropic Transformation for the Perfect Gas

The ideal closed cycle is realised with ideal machines: adiabatic and isentropic compressors and turbines, heat exchangers without pressure or heat losses. The thermodynamic calculations are especially simplified if we assume that the working fluid is a perfect gas. By perfect gas, we mean a gas that meets the following conditions:

$$\begin{aligned}\frac{P}{\rho} &= \frac{R}{M}T && \text{volumetric equation of state} \\ C_P &= C_V + R && \text{with } C_V \text{ constant} \\ \gamma &= \frac{C_P}{C_V}\end{aligned}$$

with  $M$  as the molar mass of the gas and  $R$  the universal constant of the gases. The specific enthalpy and the specific entropy, therefore, are

$$\begin{aligned}H(T) - H_0 &= C_P (T - T_0) \\ S(T, P) - S_0 &= C_P \ln \frac{T}{T_0} - R \ln \frac{P}{P_0}\end{aligned}$$

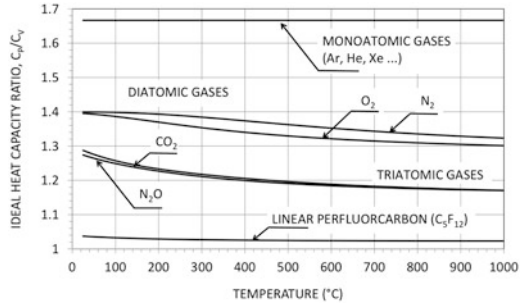
The perfect gas model approximates reasonably well to that of the ideal gas, provided the temperature variations are not extreme. For the ideal gas, the heating during isentropic compression is  $(\partial T / \partial P)_S = RT / C_P$  or

$$\frac{dT}{T} = \frac{\gamma - 1}{\gamma} \frac{dP}{P}$$

---

<sup>25</sup>However, it should be noted that even in open gas cycles, in a combined cycle layout, the exchanger for waste heat release is, in fact, present: the recovery boiler.

**Fig. 1.30** Ideal heat capacity ratio for some gases with different molecular complexity as a function of the temperature



and, in the hypothesis of a perfect gas,

$$\ln \frac{T_{2'}}{T_1} = \frac{\gamma - 1}{\gamma} \ln \frac{P_2}{P_1}$$

or

$$\frac{T_{2'}}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}} \quad (1.23)$$

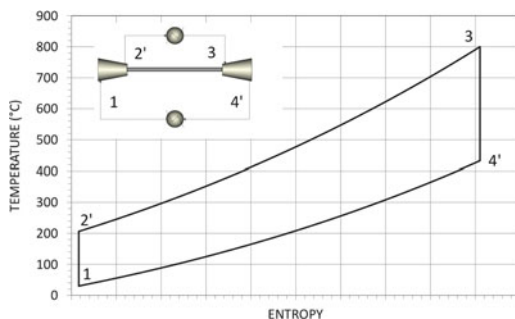
with  $T_1$  and  $P_1$  representing the temperature and pressure at the start of compression and  $T_{2'}$  and  $P_2$  the temperature and pressure at the end of isentropic compression. The ratio  $P_2/P_1 = r > 1$  is the compression ratio. For an expansion

$$\frac{T_{4'}}{T_3} = \left( \frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} = \left( \frac{1}{r} \right)^{\frac{\gamma-1}{\gamma}} \quad (1.24)$$

with  $T_{4'}$  representing the temperature at the end of expansion at pressure  $P_4$ , whilst expansion begins from temperature  $T_3$  and pressure  $P_3$ . The ratio  $P_3/P_4 = r > 1$  is the expansion ratio.

In general, for an ideal gas, the specific molar heat capacity  $C_V$  rises with the temperature and (with the temperature fixed) increases with the number of atoms composing the gas molecule. In fact, as the number of atoms present in the molecule rises, so does the degree of freedom available (in particular, the rotational modes and, above all, the vibrational ones). As the specific molar heat capacity  $C_V$  rises, the ratio  $\gamma = (C_V + R)/C_V = 1 + R/C_V$  (with the temperature fixed) tends to decrease, reaching a limit (for high molecular complexity gas) of practically unit values (see Fig. 1.30). For a ideal gas, the ratio between the specific heats  $\gamma = C_P/C_V$  depends on the temperature, but with reference to the mean values of  $C_V$  and  $\gamma$ , the above considerations are still valid.

**Fig. 1.31** Ideal closed gas cycle with ideal gas in the thermodynamic plane T-S and diagram of the plant



The high specific molar heat (which usually corresponds to a specific heat with near-normal values) leads directly to a peculiarity of molecular complex gases in their isentropic transformations. In fact, from the ratios (1.23) and (1.24), we deduce that the isentropic transformation, if  $\gamma$  approaches the unit value, also tends to become isothermal. For example, for  $C_p = 200 \text{ kJ/kmol K}$ , as can be found in certain types of fluorocarbons, we have  $\gamma = 1.043$  and  $(\gamma - 1)/\gamma = 0.042$ , which, for a pressure ratio of 5, corresponds to a temperature ratio of 1.069. For the air and helium, respectively, we get  $\gamma = 1.4$  and  $\gamma = 1.67$ ; and with a pressure ratio of 5,  $T_{2'}/T_1 = 1.584$  and  $T_{2'}/T_1 = 1.904$ . The temperature of the fluorocarbon rises by about 7 %, that of the air by 58 % and that of the helium by 90 %.

In conclusion, if we want to have equal temperature rises, we need to impose very different compression ratios on the various different fluids.

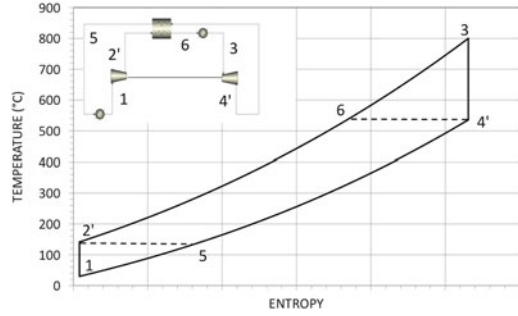
As we shall see below, in order to get a high efficiency from air and helium gas cycles, the percentage increase of the compressor temperature (or the percentage decrease of the turbine temperature) should be approximately the same. To obtain such an increase, the compression ratio of the air cycle must be considerably higher than that of the helium cycle, with important technical consequences.

### 1.7.2 The Thermodynamics of the Ideal Closed Gas Cycle

The simple ideal closed cycle, with ideal gas, is shown in Fig. 1.31 in the thermodynamic plane temperature–entropy. The gas at the minimum cycle temperature  $T_1$  is compressed from point 1 until reaching pressure  $P_2$  at point 2'. During the adiabatic (and ideal) compression, the gas is heated up to temperature  $T_{2'}$ . The compression ratio  $r = P_2/P_1$  is equal to the expansion ratio  $P_3/P_4$ . In the heater, the gas receives thermal energy at a constant pressure and reaches the maximum design temperature  $T_3$ . At this point, the gas expands from the maximum pressure  $P_3 = P_2$  to the minimum pressure  $P_4 = P_1$ , cooling down to temperature  $T_{4'}$ . The radiator then cools the gas further to the minimum temperature  $T_1$ , closing the cycle.



**Fig. 1.32** Ideal closed gas cycle with ideal gas and with a recuperative heat exchanger (the regenerator). Thermodynamic cycle in the thermodynamic plane T–S and a scheme of the plant



The efficiency of the simple ideal cycle with perfect gas can be calculated as

$$\begin{aligned}
 \eta &= \frac{W_T - W_C}{Q_{in}} & (1.25) \\
 &= \frac{(T_3 - T_{4'}) - (T_{2'} - T_1)}{(T_3 - T_{2'})} \\
 &= \frac{(T_3 - T_{2'}) - (T_{4'} - T_1)}{(T_3 - T_{2'})} \\
 &= 1 - \frac{T_1 (T_{4'}/T_1 - 1)}{T_{2'} (T_3/T_2 - T_1)} \\
 &= 1 - \frac{T_1}{T_{2'}} \\
 &= 1 - \left( \frac{1}{r} \right)^{(\gamma-1)/\gamma}
 \end{aligned}$$

since  $T_{4'}/T_1 = (T_{4'}/T_1)(T_3/T_3)(T_{2'}/T_{2'}) = T_3/T_2$ .

For compression ratios  $r$  lower than the value

$$r_{\max}^{(\gamma-1)/\gamma} = \sqrt{\frac{T_3}{T_1}} = \sqrt{\tau} \quad (1.26)$$

the temperature  $T_{4'}$  is always greater than the temperature at compression end  $T_{2'}$ , and it is possible to insert a recuperative heat exchanger, known as a regenerator,<sup>26</sup> into the cycle in order to improve the thermodynamic efficiency of the cycle, according to the diagram in Fig. 1.32.

<sup>26</sup>In reality, by “recuperative” heat exchanger, we mean an exchanger in which the two fluids (the hot one and the colder one) are physically separated by solid walls and their flow is continuous. By the term “regenerative” heat exchanger, we mean a heat exchanger made with a porous matrix, through which two fluids flow alternatively (namely, the hot one and the cold one).

For an ideal closed cycle with regeneration and a perfect gas,

$$\begin{aligned}
 \eta &= \frac{W_T - W_C}{Q_{\text{in}}} \\
 &= \frac{(T_3 - T_{4'}) - (T_{2'} - T_1)}{(T_3 - T_6)} \\
 &= \frac{T_3 \left[ 1 - (1/r)^{(\gamma-1)/\gamma} \right] - T_1 \left[ r^{(\gamma-1)/\gamma} - 1 \right]}{T_3 \left[ 1 - (1/r)^{(\gamma-1)/\gamma} \right]} \\
 &= 1 - \frac{r^{(\gamma-1)/\gamma}}{\tau}
 \end{aligned} \tag{1.27}$$

since  $T_6 = T_{4'}$  (see Sect. 1.7.4).

Where the compression ratio satisfies the ratio (1.26), the efficiency expressed by (1.25) is equal to that calculated by (1.27).

The useful work for the cycle (the same for the ideal cycle with regeneration and for the simple ideal cycle) is

$$\frac{W}{C_p T_1} = \tau \left[ 1 - \left( \frac{1}{r} \right)^{(\gamma-1)/\gamma} \right] - \left[ r^{(\gamma-1)/\gamma} - 1 \right] \tag{1.28}$$

Introducing the parameter  $\psi = (T_{2'} - T_1) / T_1 = (r^{(\gamma-1)/\gamma} - 1)$ , which represents the temperature rise in the compressor, (1.27) and (1.28) become

$$\eta = 1 - \frac{1 + \psi}{\tau} \tag{1.29}$$

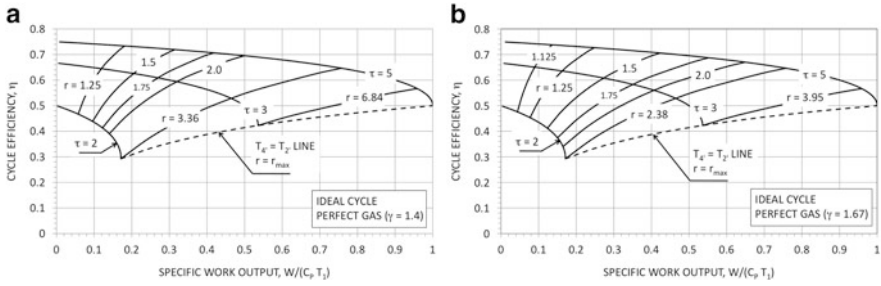
$$\frac{W}{C_p T_1} = \tau \frac{\psi}{1 + \psi} - \psi \tag{1.30}$$

Here below, we shall consider just the regenerative cycle. The closed gas cycle is, in fact, always the regenerative type, largely because some heat must be released into the environment by means of a heat exchanger, and the use of this regenerator simply allows this heat release to be divided into two phases, obtaining, at the same time, a significant preheating of the working fluid on entry into the heater and, contemporarily, a decrease in the waste heat, which gives the cycle an increased thermodynamic efficiency.

The choice of the working gas for the cycle has a direct influence on the thermodynamic characteristics of the cycle and on the size of the used machinery.

### *Influence of the Working Fluid on Efficiency and on Useful Work in the Cycle*

The two equations above (1.29) and (1.30) lead us to deduce that any working fluid (in the hypothesis, this is assumed to be a perfect gas and the machinery used ideal)



**Fig. 1.33** Cycle efficiency and specific dimensionless work for ideal gas cycles with perfect gas

gives the same performance,  $\tau$  and  $\psi$  being equal. What changes as the gas varies and  $\tau$  remains fixed is the compression ratio necessary to obtain the desired value of  $\psi$  (see Sect. 1.7.1). The value of the specific useful work  $W = W_T - W_C$  will then depend on the value  $T_1$  and, to a large extent, on the type of gas used, through the specific heat  $C_p$ . With a fixed  $\tau$  value, the cycle efficiency (in the case of an ideal cycle) diminishes with  $\psi$ , whilst the specific work output  $W/(C_p T_1)$  reaches optimal where  $\psi = \sqrt{\tau} - 1$ , or in correspondence with the compression ratio  $r = r_{\max}$  of (1.26).

Figure 1.33 represents, for an ideal gas cycle with regeneration, the efficiency for the cycles and specific work  $W/(C_p T_1)$  of two perfect gases (one with  $\gamma = 1.4$  and the second with  $\gamma = 1.67$ ). Fixed  $\tau$ , the maximum work is obtained with the compression ratio  $r_{\max}$ , when the cycle efficiency is at its minimum value. So, closed gas cycles with regeneration are generally characterised by modest compression ratios (necessary for good performances) and, consequently, by relatively modest specific work values. The effect of the molecular complexity of the working fluid is highlighted, in Fig. 1.33, by the different values of  $r_{\max}$ . For example, fixed  $\tau = 3$  gives  $r_{\max} = 6.84$  in the case of the fluid with  $\gamma = 1.4$  (air) and 3.95 for the fluid with  $\gamma = 1.67$  (monoatomic gas), in confirmation of what was said in Sect. 1.7.1.

### *Influence of the Working Fluid on the Size of the Rotating Machinery*

Referring to Eqs. (1.29) and (1.30), we can see how the expansion work

$$\frac{W_T}{C_p T_1} = \tau \frac{\psi}{1 + \psi}$$

and the compression work

$$\frac{W_C}{C_p T_1} = \psi$$

are just functions of the ratios  $\psi$  and  $\tau$ . With reference to the turbine, considering two distinct perfect gases (of which one of them, the reference, is identified by the index “0”), with fixed minimum temperature  $T_1$ ,  $\tau$  ratio,  $\psi$  ratio and the work per stage  $(\Delta H_j)_S$ , we get a ratio between the number of stages necessary equal to

$$\frac{N_T}{N_{T,0}} = \frac{\gamma}{\gamma - 1} \frac{\gamma_0 - 1}{\gamma_0} \frac{M_0}{M}$$

with  $M$  representing the molar mass of the working fluid. For example, the number of stages is about five times greater for helium than for air.

The work per stage (or the total enthalpy drop over the average stage, which is also equal to the difference in the static enthalpy  $(\Delta H_j)_S$  if the stages are repeated and ideal) and the isentropic volume flow rate  $(\dot{V}_{out,j})_S$  at the stage outlet define the so-called specific speed  $\omega_s$ :

$$\omega_s = \frac{2\pi N}{60} \frac{\sqrt{(\dot{V}_{out,j})_S}}{(\Delta H_j)_S^{3/4}}$$

with  $N$  being the number of revs. The specific speed may be optimised and the dimensional analysis guarantees that (once optimised) the results for one turbine stage can be extended to all the machines with the same  $\omega_s$  parameter, provided that these are working with fluids that have the same thermodynamic behaviour, that the geometric similarity is strictly respected and that the effect of the Reynolds number can be considered negligible.

The variation in the volume flow rate during expansion means that the  $\omega_s$  values may vary considerably from one stage to another: smaller for the high-pressure stages and larger for low-pressure stages. Assuming that  $(\Delta H_j)_S$  is the same for all stages, we calculate the  $\omega_s$  variation between the first and last stage of a turbine operating with a perfect gas,

$$\begin{aligned} \frac{\omega_{s,N_T}}{\omega_{s,1}} &= \sqrt{\frac{(\dot{V}_{out,N_T})_S}{(\dot{V}_{out,1})_S}} \\ &= \sqrt{\frac{(\rho_{out,1})_S}{(\rho_{out,N_T})_S}} \end{aligned}$$

The density for a perfect gas is calculated, once the pressure and temperature have been found from the volumetric equation of state  $\rho = PM/RT$ . For the final stage of the turbine,  $T = T_{4'}$  and  $P = P_4$  and  $(\rho_{out,N_T})_S = P_4 M / RT_{4'}$ . For the first stage,

$$\begin{aligned}
T &= T_3 - \frac{(\Delta H_j)_S}{C_P} \\
&= T_3 \left( 1 - \frac{W_T}{C_P T_1} \frac{1}{N_T} \frac{T_1}{T_3} \right) \\
&= T_3 \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right) \\
P &= P_3 \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right)^{\frac{\gamma}{\gamma-1}} \\
(\rho_{\text{out},1})_S &= \frac{P_3 M}{R T_3} \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

Then

$$\begin{aligned}
\frac{\omega_{s,N_T}}{\omega_{s,1}} &= \sqrt{\frac{(\rho_{\text{out},1})_S}{(\rho_{\text{out},N_T})_S}} \\
&= \sqrt{\frac{P_3}{P_4} \frac{T_4}{T_3} \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right)^{\frac{1}{\gamma-1}}} \\
&= \sqrt{\left( \frac{T_3}{T_4} \right)^{\frac{1}{\gamma-1}} \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right)^{\frac{1}{\gamma-1}}} \\
&= \sqrt{(1 + \psi)^{\frac{1}{\gamma-1}} \left( 1 - \frac{\psi}{1 + \psi} \frac{1}{N_T} \right)^{\frac{1}{\gamma-1}}}
\end{aligned}$$

If  $N_T$  (the number of stages) is sufficiently high,  $\omega_{s,N_T}/\omega_{s,1} \approx \omega_{s,\infty}/\omega_{s,1} = \lim_{N_T \rightarrow +\infty} \omega_{s,N_T}/\omega_{s,1} = \sqrt{(1 + \psi)^{\frac{1}{\gamma-1}}}$ . For  $\psi = 0.4$  (corresponding to a compression ratio  $r = 4.3$  for a perfect gas with  $\gamma = 1.3$ , at  $r = 3.25$  for a perfect gas with  $\gamma = 1.4$  and at a compression ratio  $r = 2.31$  for a perfect gas with  $\gamma = 1.67$ ), we get the following:  $\omega_{s,\infty}/\omega_{s,1} = 1.75$  for  $\gamma = 1.3$  (triatomic gas, carbon dioxide), 1.52 for  $\gamma = 1.4$  (diatomic gas, nitrogen), and 1.28 for  $\gamma = 1.67$  (monoatomic gas, helium).

The flow areas at the final-stage outlet  $A_{\text{out},N_T}$  and at the first-stage outlet  $A_{\text{out},1}$  are (for similar axial speed of the gas) in the ratio

$$\frac{A_{\text{out},N_T}}{A_{\text{out},1}} = \left( \frac{\omega_{s,N_T}}{\omega_{s,1}} \right)^2$$

and with reference to the previous example,  $A_{\text{out},N_T}/A_{\text{out},1} = 3$  for  $\gamma = 1.3$ , 2.32 for  $\gamma = 1.4$  and 1.65 for  $\gamma = 1.67$ . The ratio between the mean diameter and the height of the blades for the final and for the first stage varies with the square root of

the flow areas, that is, they are in ratio to the specific speeds. From the examples we considered, it emerges that, in the case of monoatomic perfect gas, the turbine has more limited variations in the radial dimension than with the triatomic gas (about 30 % less, for the same turbine power).

At similar turbine power and with the maximum pressure and temperature  $P_3$  and  $T_3$  fixed, it is possible to calculate the ratio between the number of revs of two turbines operating with two different fluids (of which one is the reference and is identified by the index “0”), assuming the peripheral speed is the same. With reference to the first stage,

$$\begin{aligned}
 \frac{A_{\text{out},1}}{A_{\text{out},1,0}} &= \frac{\gamma_0}{\gamma_0 - 1} \frac{\gamma - 1}{\gamma} \frac{M}{M_0} \frac{(\rho_{\text{out},1,0})_S}{(\rho_{\text{out},1})_S} \\
 &= \frac{\gamma_0}{\gamma_0 - 1} \frac{\gamma - 1}{\gamma} \frac{\left(1 - \frac{\psi}{1+\psi} \frac{1}{N_{T,0}}\right)^{\frac{1}{\gamma_0-1}}}{\left(1 - \frac{\psi}{1+\psi} \frac{1}{N_T}\right)^{\frac{1}{\gamma-1}}} \\
 \frac{N}{N_0} &= \sqrt{\frac{A_{\text{out},1,0}}{A_{\text{out},1}}} \\
 &= \sqrt{\frac{\gamma}{\gamma - 1} \frac{\gamma_0 - 1}{\gamma_0}} \sqrt{\frac{\left(1 - \frac{\psi}{1+\psi} \frac{1}{N_T}\right)^{\frac{1}{\gamma-1}}}{\left(1 - \frac{\psi}{1+\psi} \frac{1}{N_{T,0}}\right)^{\frac{1}{\gamma_0-1}}}}
 \end{aligned}$$

If  $N_T$  and  $N_{T,0}$  (the number of stages for the two turbines) are sufficiently high,  $N/N_0 \approx N_\infty/N_{\infty,0} = \lim_{N_T \rightarrow +\infty} N/N_0 = \sqrt{\frac{\gamma}{\gamma-1} \frac{\gamma_0-1}{\gamma_0}}$ . For  $\gamma_0 = 1.4$  and  $\gamma = 1.67$ , we get  $N_\infty/N_{\infty,0} = 0.84$ . For similar useful power and at the same maximum pressure and temperature, the turbine of a closed helium cycle has a number of revolutions which is 15 % lower than the corresponding turbine with nitrogen.

So far, our considerations have concerned ideal cycles operating with perfect gas. Whilst the qualitative conclusions regarding the characteristics of the turbomachinery are valid in general, the cycle efficiency will change considerably when we introduce the real machine efficiencies. For this reason, the evaluation of the real cycle performances must take into consideration: the efficiency of the machines that they contain, which are not unitary; the pressure losses, which are always present; as well as the effective thermodynamic characteristics of the working fluid (in particular, the specific heat) which, if the gas is ideal, are a function of the temperature. The power necessary at the compressor is transmitted directly by the turbine via a shaft, which introduces a loss that is taken into account by means of the  $\eta_m$  mechanical efficiency (assuming it to be constant,  $\eta_m = 0.95$ ).

A gas cycle is extremely sensitive to pressure losses on the heat exchangers (and, generally speaking, to all losses) because the ratio  $W/(W_T + W_C)$  is fairly modest

(in the case of an ideal cycle,  $W/(W_T + W_C) = [1 - (1 + \psi)/\tau] / [1 + (1 + \psi)/\tau]$  is 0.4, for  $\psi = 0.4$  and  $\tau = 3.21$ ). Therefore, it is imperative that the losses be kept to a minimum that is technologically and economically reasonable. As far as pressure losses are concerned, the thermodynamic irreversibility that they introduce, which directly penalises the efficiency, is comparable to a throttling process, and if the gas is a perfect gas, for the purposes of calculating the cycle efficiency, it has no importance where it appears. In the following examples, for the sake of simplicity, all the pressure losses will be concentrated at the compressor outlet, and their net effect will be that of reducing the expansion ratio, which will be lower than the compression ratio ( $r_T = P_3/P_4 < r_C = P_2/P_1$ ). The pressure losses will be introduced as a fraction of the maximum pressure (the pressure at the discharge of the compressor). In this way,  $P_3 = P_2(1 - \Delta P/P_2)$  and  $r_T = r_C(1 - \Delta P/P_2)$ .

In Sects. 1.7.3 and 1.7.4, we shall discuss the definitions of compression, expansion, and regenerator efficiency.

### 1.7.3 Compressor and Turbine Efficiency

With regard to thermal turbomachinery, in order to calculate the work of compression and expansion, we usually refer to the isentropic (or adiabatic) efficiency, defined as

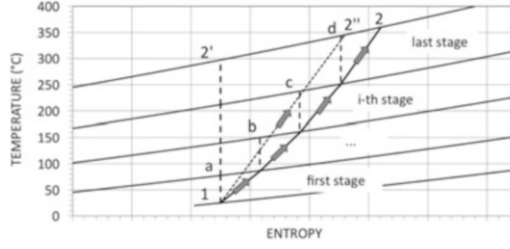
$$\eta_C = \frac{(\Delta H)_S}{\Delta H} \quad \text{for a compression} \quad (1.31a)$$

$$\eta_T = \frac{\Delta H}{(\Delta H)_S} \quad \text{for an expansion} \quad (1.31b)$$

where  $\Delta H$  represents the real work and  $(\Delta H)_S$  the work corresponding to an ideal adiabatic transformation (an isentropic one). The two efficiencies  $\eta_C$  and  $\eta_T$  are an index of the thermo-fluid dynamic efficiency which characterises the transformation, with the definition taking into consideration the fact that, by its very nature, the turbomachinery is poorly adapted to the continuous cooling or reheating of the working fluid.<sup>27</sup> Equations (1.31a) and (1.31b) are easy to apply in the case of a perfect gas because the isentropic work is easily calculated. In fact,

---

<sup>27</sup>In fact, there is very limited surface space available inside them for heat exchange to take place, and the fluid touches them at such high speed. For this reason, in the case of compressors, when seeking to reduce the average temperatures of the transformation, the compression is interrupted once the fluid has undergone a significant reheating; the gas is sent to a heat exchanger, where its temperature is brought back to values similar to the start; the compression is then completed at a second stage (intercooled compression). Naturally, there may be more than one intercooling stage.



**Fig. 1.34** Temperature variation during compression in the case of (i) isentropic compression from point 1 to point 2', (ii) adiabatic compression achieved with a single stage of fixed efficiency  $\eta_C$  from point 1 to point 2'', (iii) adiabatic compression achieved with four stages, each with an efficiency  $\eta_C$  from point 1 to the final point 2

$$\eta_C = \frac{T_1 \left( r_C^{(\gamma-1)/\gamma} - 1 \right)}{T_2 - T_1} \quad \text{for a compression} \quad (1.32a)$$

$$\eta_T = \frac{T_3 - T_4}{T_3 \left( 1 - \left( \frac{1}{r_T} \right)^{(\gamma-1)/\gamma} \right)} \quad \text{for an expansion} \quad (1.32b)$$

and, therefore, it is simple, given that we know  $\eta_C$  and  $\eta_T$ , to evaluate the temperatures  $T_2$  and  $T_4$  at the end of the transformation and the corresponding work of compression  $C_P (T_2 - T_1)$  and of expansion  $C_P (T_3 - T_4)$ .

In reality, compression and expansion are divided up and carried out via a highly variable number of stages (in the case of traditional gas cycles, from a minimum of 2–3 compression stages and 2–3 turbine stages up to 20 compression stages and 7–10 expansion stages; with radial or axial machines, this will depend on the power and the compression/expansion ratios). In such cases, by associating each stage with an isentropic efficiency and assuming that it is the same for all the stages, we obtain—with reference to compression—that which is illustrated in Fig. 1.34.

On account of the fluid dynamic losses of the preceding stages, the typical compression stage has to work with a fluid that has a higher average temperature than it would if those previous stages had been ideal. The result is an effective compression efficiency  $\eta_{C,N_C}$  which, with the fixed compression ratio and the efficiency of each individual stage, becomes a function of the number of stages  $N_C$ :

$$\begin{aligned} \eta_{C,N_C} &= \frac{(\Delta H)_S}{\sum_{j=1}^{N_C} \Delta H_j} \\ &= \eta_C \frac{(\Delta H)_S}{\sum_{j=1}^{N_C} (\Delta H_j)_S} \end{aligned}$$

Since  $(\Delta H)_S < \sum_{j=1}^{N_C} (\Delta H_j)_S$ , the efficiency  $\eta_{C,N_C}$  is lower than the efficiency  $\eta_C$  (a negative consequence of an unwanted “preheat” of the gas during the compression).



Where the expansion is broken up into  $N_T$  stages, there is a similar phenomenon, and we get

$$\begin{aligned}\eta_{T,N_T} &= \frac{\sum_{j=1}^{N_T} \Delta H_j}{(\Delta H)_S} \\ &= \eta_T \frac{\sum_{j=1}^{N_T} (\Delta H_j)_S}{(\Delta H)_S}\end{aligned}$$

In such a case, the efficiency  $\eta_{T,N_T}$  is greater than the efficiency  $\eta_T$  (as the “reheating” in one stage is partially recovered in the next one).

As a result, the isentropic efficiencies  $\eta_C$  and  $\eta_T$  do not really take into account the real thermodynamic quality of the overall transformation (although, at least for a perfect gas, they are quick and easy to apply), so preference is given to the polytropic efficiency, defined as the (local) efficiency of an infinitesimal compression or expansion (i.e. breaking the compression or expansion up into infinite stages):

$$\eta_{C,\infty} = \frac{dP/\rho}{dH} \quad \text{for a compression} \quad (1.33a)$$

$$\eta_{T,\infty} = \frac{dH}{dP/\rho} \quad \text{for an expansion} \quad (1.33b)$$

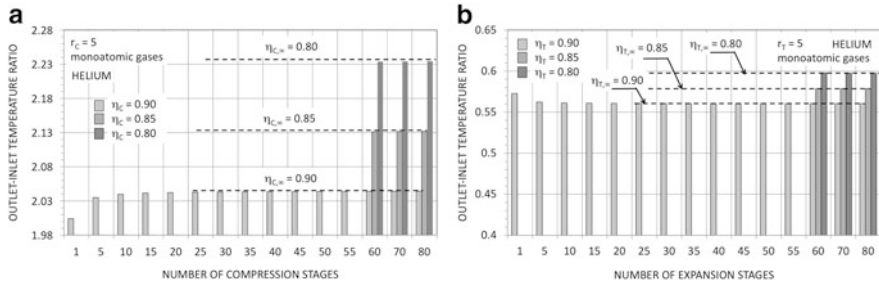
In the case of a perfect gas, the integration of (1.33a) and (1.33b) is immediate and gives

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{(\gamma-1)/\eta_{C,\infty}\gamma} \quad \text{for a compression} \quad (1.34a)$$

$$\frac{T_4}{T_3} = \left( \frac{P_4}{P_3} \right)^{\eta_{T,\infty}(\gamma-1)/\gamma} \quad \text{for an expansion} \quad (1.34b)$$

In general, the specific heat of those gases comparable to ideal gases is a function of the temperature (only monoatomic gases are completely comparable to a perfect gas), whilst with a real gas, there are also effects linked to the volumetric behaviour. In such cases, integrating (1.33a) and (1.33b) is not usually simple, and the problem arises of the number of stages used in the preliminary cycle calculations in order to get expansions and compressions that have a polytropic efficiency equal to that which was assumed. In fact, the number and type of stages to be employed are not known a priori, as this is connected to the kind of fluid used, the compression ratios, the level of power, etc.

By using similitude and dimensional analysis, it is possible to carry out a thermo-fluid dynamic cycle optimisation, using appropriate calculation codes and taking into account the effective efficiency of each stage of the compressor and



**Fig. 1.35** Temperature ratio for an adiabatic compression (a) and expansion (b) as the number of stages varies and for different efficiency values. For purposes of clarity, for the cases  $\eta_C = \eta_T = 0.85$  and  $0.8$ , the figure only shows the results relative to 60, 70, and 80 stages

turbine; in a purely thermodynamic preliminary analysis, it may be enough to break up the transformation into a sufficiently high number of stages to approximate the efficiency  $\eta_{C,N_C}$  and  $\eta_{T,N_T}$  at the values  $\eta_{C,\infty}$  and  $\eta_{T,\infty}$ .

Figure 1.35 reports the values  $T_2/T_1$  and  $T_4/T_3$  as the number of compression and expansion stages varies (with reference to helium, for  $r_C = r_T = 5$  and for three different values of isentropic and polytropic efficiency). As the number of stages grows, the temperature ratio tends to correspond to a value corresponding to those in (1.34a) and (1.34b). When the number of stages reaches a few dozen, the difference is well below 1 %. In the cycle examples illustrated below, we assume the number of stages to be 80.

### 1.7.4 Efficiency of the Recuperator and the Effect of the Pressure on the Design of Heat Exchangers

The recuperative heat exchanger (see Fig. 1.32, valid for the ideal cycle) enables the gas originating from the compressor to be preheated by the sensible heat of the same gas (hotter) at the turbine outlet. The recuperative heat exchanger, aka regenerator, is characterised by a heat transfer effectiveness, defined as the ratio between the thermal power really exchanged and the thermal power that would be exchanged if the surfaces of the exchanger were infinite:

$$\begin{aligned}
 \epsilon_R &= \frac{\dot{Q}_R}{\dot{Q}_{\max,R}} \\
 &= \frac{\dot{m} (H_6 - H_2)}{\dot{m} (H_4 - H_{5'})} \\
 &= \frac{(H_6 - H_2)}{(H_4 - H_{5'})}
 \end{aligned} \tag{1.35}$$

with  $H_2$  enthalpy of the gas at the exit of the compressor (in the case of real compression),  $H_4$  gas enthalpy at the turbine discharge (in the case of real expansion) and  $H_6$  the enthalpy of the heated gas (on HP side) at the end of the regeneration. The enthalpy  $H_{5'}$  is the enthalpy of the gas at the temperature  $T_2$  but on the LP side of the regenerator.

If the gas is comparable to a perfect gas,

$$\epsilon_R = \frac{T_6 - T_2}{T_4 - T_2} \quad (1.36)$$

If  $\epsilon_R = 1$ , then  $T_6 = T_2$ . For  $\epsilon_R < 1$

$$T_6 = T_2 + \epsilon_R (T_4 - T_2) \quad (1.37)$$

and the temperature difference  $T_6 - T_4 = (1 - \epsilon_R)(T_2 - T_4) = T_2 - T_5$  is the minimum temperature difference reached in the exchanger, and it is constant throughout the exchanger: the two isobars  $P_2$  and  $P_4$ , in the  $T - \dot{Q}$  plane, are shown to be parallel.

It is useful to employ (1.35) (or version (1.36) for a perfect gas) in making the thermodynamic calculations for the cycle; it shows that (1.35), in the case of monophase fluids, can be expressed as a function of two nondimensional parameters and of the flow arrangement (counterflow, parallel flow, crossflow, multi-pass, periodic flow, etc.) of the heat exchanger (see [26]):

$$\epsilon_R = \epsilon(\text{NTU}_{\max}, C_{\min}/C_{\max}, \text{flow arrangement}) \quad (1.38)$$

with  $C_{\min}$  and  $C_{\max}$ , respectively, as the minimum and maximum value between  $\dot{m}_{\text{cold}}C_{P,\text{cold}}$  and  $\dot{m}_{\text{hot}}C_{P,\text{hot}}$ . The coefficient  $\text{NTU}_{\max} = AU/C_{\min}$  is the number of transfer units ( $A$  is the heat exchanger global area and  $U$  is the overall heat transfer coefficient).

Equation (1.38) has different analytical forms according to the type of exchanger and becomes useful whenever it is necessary to select the size of the exchanger, once the overall heat transfer coefficient has been calculated.

### *Influence of the Working Fluid on the Size of the Recuperator*

The heat recuperator is a surface heat exchanger which is normally made with compact heat exchanger surfaces. Whatever its geometry, the heat transfer and the fluid friction characteristics of the heat exchange matrix can be represented qualitatively by means of equations like (see [27])

$$\text{St}Pr^{2/3} = \phi_h(Re) \quad (1.39a)$$

$$f = \phi_f(Re) \quad (1.39b)$$

where  $St = h/GC_P$  is the Stanton number,  $Pr = \mu C_P/k$  is the Prandtl number and  $Re = 4r_h G/\mu$  is the Reynolds number<sup>28</sup> and  $\phi_h(Re)$  and  $\phi_f(Re)$  are two functions of the Reynolds number, dependent on the type of heat exchange matrix (e.g. tube-bank surfaces with plain fins, plain plait with fin surfaces)

From (1.39a) and the definition of the Stanton number  $St$ , we obtain the heat transfer coefficient  $h$ :

$$h = \frac{\mu C_P}{Pr^{2/3}} \frac{\phi_h}{4r_h} Re$$

The thermal power  $\dot{Q}$  exchanged by the fluid may be calculated as

$$\dot{Q} = hA\Lambda = \frac{\mu C_P}{Pr^{2/3}} \frac{\phi_h}{4r_h} Re A \Lambda$$

where  $\Lambda$  is the logarithmic mean temperature difference in the heat exchanger.

The friction power  $\dot{W}_f$  can be calculated as

$$\dot{W}_f = \frac{\Delta P}{\rho} G A_c = \left( \frac{f}{2} \frac{G^2}{\rho^2} \frac{A}{A_c} \right) G A_c$$

or using (1.39b) and rearranging the resulting equation,

$$\dot{W}_f = \frac{1}{2} \frac{\mu^3}{\rho^2} \left( \frac{1}{4r_h} \right)^3 Re^3 \phi_f A$$

At this point, the ratio  $\dot{W}_f/\dot{Q}$ , which represents the ratio between the mechanical power necessary for the circulation of the gas and the thermal power exchanged, is

$$\frac{\dot{W}_f}{\dot{Q}} = \frac{1}{2} \frac{\phi_f}{\phi_h} \left( \frac{v^2}{\Lambda} \right) \left( \frac{Pr^{2/3}}{C_P} \right) \propto \left( \frac{v^2}{\Lambda} \right) \left( \frac{1}{C_P} \right) \quad (1.40)$$

where  $v$  is the velocity of the gas and the Prandtl number  $Pr \approx 1$  for the gases.

The ratio  $\dot{W}_f/\dot{Q}$  is proportional to the product of two terms: the first takes into account the working conditions (the fluid velocity and the logarithmic mean temperature difference in the heat exchanger), and the second depends on the kind of fluid used by means of its specific heat  $C_P$ . Therefore, we conclude that a low value for the ratio  $\dot{W}_f/\dot{Q}$  can be obtained by limiting the speed  $v$ , adopting high logarithmic mean temperature differences and choosing a gas with a high specific heat at constant pressure.

---

<sup>28</sup>The parameters that define the adimensional numbers in question are  $4r_h = 4A_c L/A$ ,  $G = \dot{m}/A_c$ , with  $r_h$  being the hydraulic radius,  $A_c$  the minimum total free-flow cross-sectional area,  $L$  the length of tubes and  $A$  the total surface area.  $G$  is the mass velocity. The coefficient  $h$  is the heat transfer coefficient,  $\mu$  is the viscosity and  $k$  is the thermal conductivity of the gas considered.

Equation (1.40) can be further elaborated, eliminating the speed  $v$  of the gas (see [28]), considering that the thermal heat exchanged  $\dot{Q}$  can also be calculated as

$$\dot{Q} = \dot{m} C_p \Delta T = (G A_c) C_p \Delta T$$

with  $\Delta T$  as the temperature variation of the fluid. If we define the volume of the exchanger  $V = A_{fr} L$ , with  $A_{fr}$  being the frontal area and  $L$  the length of the heat exchanger, taking into account that the frontal area  $A_{fr}$  and the free frontal area  $A_c$  are related via the free-to-frontal area ratio  $\sigma = A_c/A_{fr}$ , multiplying and dividing the second term of (1.40) by  $\dot{Q}$  and by  $V$  (the volume of the exchanger), we get

$$\frac{\dot{W}_f}{\dot{Q}} = \frac{1}{2} \frac{\phi_f}{\phi_h} \left( \frac{v^2}{\Lambda} \right) \left( \frac{Pr^{2/3}}{C_p} \right) \frac{\dot{Q}^2}{G^2 A_c^2 C_p^2 \Delta T^2} \left( \frac{A_c L}{\sigma} \right)^2 \frac{1}{V^2}$$

or

$$\frac{\dot{W}_f}{\dot{Q}} = \frac{1}{2} \frac{\phi_f}{\phi_h} \frac{\dot{Q}^2}{\Lambda \Delta T^2} \left( \frac{L}{\sigma V} \right)^2 \left( \frac{Pr^{2/3}}{\rho^2 C_p^3} \right) \quad (1.41)$$

From (1.41), it emerges that, provided the following are equal [48],

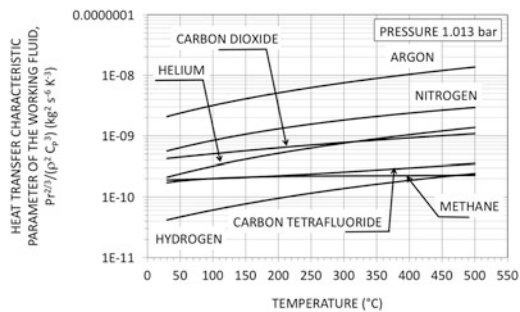
- The thermal heat exchanged (or, roughly speaking, useful power).
- Logarithmic mean temperature difference.
- Variation in fluid temperature.
- Volume occupied by the gas (and, in general terms, the cost of the exchanger, which depends almost linearly on the volume).
- The length of the exchanger to be crossed.

a good heat transfer fluid must have a low value of the ratio  $(Pr^{2/3}/(\rho^2 C_p^3))$ . An efficient heat exchanger must have a low  $L/V$  ratio, that is, a large passage section  $A_c$  compared to the length of the exchanger.

For a gas  $Pr \approx 1$  and for an ideal gas, the product  $\rho^2 C_p^3$  depends on the temperature, the pressure and on the ratio  $\gamma$  of the specific heats. We then obtain

$$\frac{\dot{W}_f}{\dot{Q}} \propto \frac{\dot{Q}^2}{\Lambda \Delta T^2} \left( \frac{L}{V} \right)^2 \left( \frac{T}{P} \right)^2 \left( \frac{M}{R} \right) \left( \frac{\gamma - 1}{\gamma} \right)^3 \quad (1.42)$$

In (1.42), the terms that depend on the operating conditions and the thermo-physical properties of the gas (which intervene by means of  $\gamma$  and the molar mass  $M$ ) have been separated. There is a clear advantage to be gained from pressurisation, since an increase in pressure leads to a significant fall in  $\dot{W}_f/\dot{Q}$ . An increase in the speed of the gas, on the other hand, would have the benefit of a better exchange coefficient, but at the cost of an increased  $\dot{W}_f/\dot{Q}$  ratio; see (1.40).



**Fig. 1.36** Heat transfer parameter  $Pr^{2/3} / (\rho^2 C_p^3)$  of various working fluids as a function of temperature at ambient pressure

**Table 1.5** Some physical properties of the working fluids of Fig. 1.36

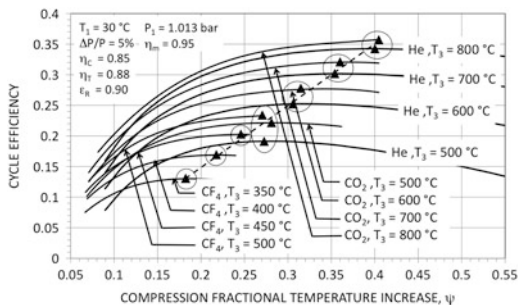
| Fluid                                 | Molecular weight | Critical temperature (K) | Critical pressure (bar) | Heat capacity ratio $C_p/C_v$ (at 30 °C, 1.013 bar) |
|---------------------------------------|------------------|--------------------------|-------------------------|-----------------------------------------------------|
| Helium, He                            | 4.003            | 5.19                     | 2.27                    | 1.68                                                |
| Argon, Ar                             | 39.948           | 150.86                   | 150.86                  | 1.68                                                |
| Xenon, Xe                             | 131.29           | 289.74                   | 58.40                   | 1.68                                                |
| Hydrogen, H <sub>2</sub>              | 2.016            | 32.98                    | 12.93                   | 1.41                                                |
| Nitrogen, N <sub>2</sub>              | 28.014           | 126.2                    | 33.98                   | 1.41                                                |
| Methane, CH <sub>4</sub>              | 16.043           | 190.56                   | 45.99                   | 1.31                                                |
| Carbon dioxide, CO <sub>2</sub>       | 44.01            | 304.12                   | 73.74                   | 1.29                                                |
| Carbon tetrafluoride, CF <sub>4</sub> | 88.005           | 227.51                   | 37.45                   | 1.16                                                |

The  $\dot{W}_t/\dot{Q}$  ratio is linked to the characteristics of the fluid via its molecular complexity and its molar mass. This favours those fluids with a low  $\gamma$ , that is, with an elevated molecular complexity and low molar mass.

Our conclusions have a general validity and indicate a tendency, emphasising both economic and technical considerations, which are linked to the operating conditions (maximum temperatures, materials, etc.) to the cost of the working fluid, to its thermal stability and to its safety for use (nonflammability, toxicity, etc.), to its environmental friendliness: in the end, reaching the right balance between plant costs and operating costs.

Figure 1.36 shows, for certain gases at ambient pressure, the values for the parameter  $\chi = Pr^{2/3} / (\rho^2 C_p^3)$  as a function of the temperature. Table 1.5 lists the molar mass, data for the critical point and the ratio  $\gamma = C_p/C_v$  for specific heats at ambient temperature and ambient pressure. It is hydrogen (with the minimum molar mass) that has the best values for the  $\chi$  parameter; methane (CH<sub>4</sub>) and carbon tetrafluoride (CF<sub>4</sub>) are very similar, carbon dioxide and helium are equivalent with temperatures over 200–250 °C and argon (a monoatomic gas with a high molar mass) is the worst of the fluids considered. As we have already observed, since  $\chi$  is inversely proportional to the square of the density, an increase in the pressure will produce a more than proportional fall in its value, giving significant benefits for the

**Fig. 1.37** Cycle efficiency for various working fluids as a function of the compression fractional temperature increase  $\psi$  for various maximum cycle temperatures  $T_3$



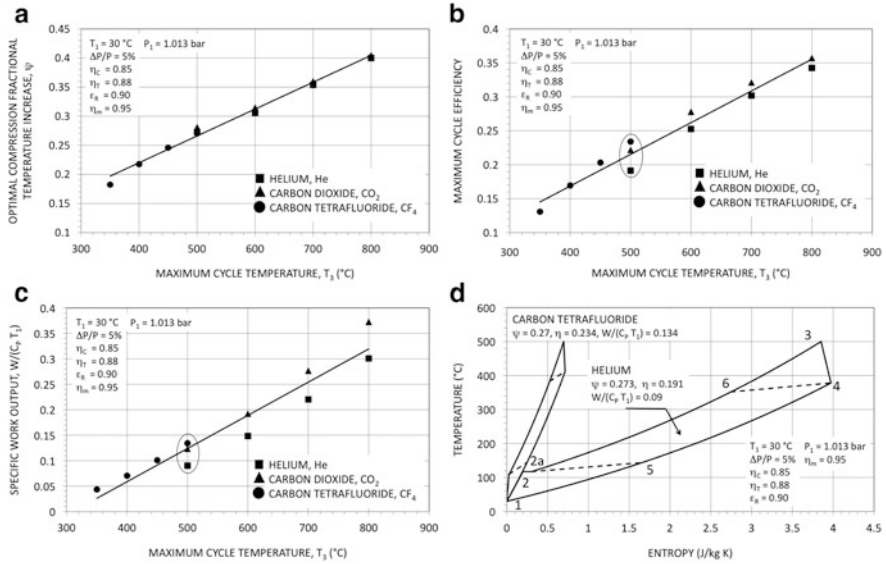
$\dot{W}_f/\dot{Q}$  ratio. In the specific case of the recuperator in a closed gas cycle, there is a great difference in the fluid density of the two branches, so the size and choice of the final geometry of the exchanger, which must obviously take into account the presence of both fluids, will generally be quite complicated.

### 1.7.5 The Thermodynamic of the Real Cycles

Figure 1.37 reports the efficiency of real cycles operating with various working fluids: helium, carbon dioxide, and carbon tetrafluoride. The carbon tetrafluoride ( $\text{CF}_4$ ) has been taken as an example of a gas with medium to high molecular complexity and molar mass (see Table 1.5). The efficiencies  $\eta_c$  and  $\eta_t$  of compression and expansion (comparable to polytrophic efficiencies, given the large number of stages considered in the calculation; see Fig. 1.35) have been assumed to be 0.85 and 0.88, respectively. The efficiency of the recuperative heat exchanger (the regenerator)  $\epsilon_R$  is 0.90. The pressure losses (5 % of the maximum pressure  $P_2$ ) have been concentrated downstream from the compressor.

According to (1.29) and (1.30), the efficiency and specific useful work, having established the ratio between the temperature extremes of the cycle, are a function of just the parameter  $\psi = (T'_2 - T_1) / T_1$ . However, these refer to the case of a perfect gas and are valid for an ideal cycle. The results in Fig. 1.37 show that, at least for the points of optimal efficiency, for the real cycles, too, there is, generally speaking, a close link between  $\psi = (T_2 - T_1) / T_1$  and the value of optimal efficiency for the cycle, irrespectively of the working fluid (having established the ratio between the temperature extremes of the cycle, the pressure losses and the machinery efficiency).

Figure 1.38a–c summarises, for a variety of working fluids and ratio of optimal compression, the fraction increase  $\psi$  of the temperature in the compressor, the cycle efficiency, and the specific work output  $W / (C_p T_1)$ . As the maximum temperature  $T_3$  increases, so does the  $\psi$  ratio of the optimal efficiency, rising from about 0.22 for  $T_3 = 400^\circ\text{C}$  to 0.4 when  $T_3$  stands at  $800^\circ\text{C}$ . Whilst the  $\psi$  values turn out to be closely related to the changes in maximum temperature, the cycle efficiency and specific work values are less so (see, e.g. cases of  $T_3 = 500^\circ\text{C}$ ): as a consequence of the varied nature of the working fluid and the varying effect of the losses.

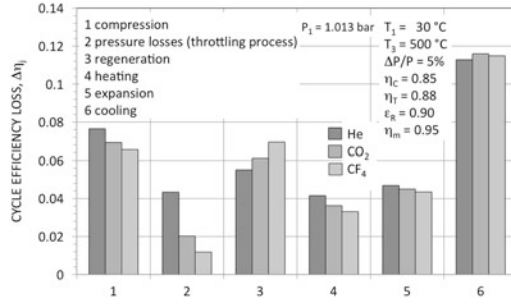


**Fig. 1.38** (a) Compression fractional temperature increase, (b) cycle efficiency, and (c) specific work output at optimum conditions at varying maximum cycle temperature. The considered working fluids are helium, carbon dioxide, and carbon tetrafluoride. (d) Two cycles at the optimum  $\psi$  with  $T_3 = 500^\circ\text{C}$  in the T-S plane. The value of  $C_p$  assumed in the calculation of the specific work output  $W/C_p T_1$  is that at  $30^\circ\text{C}$

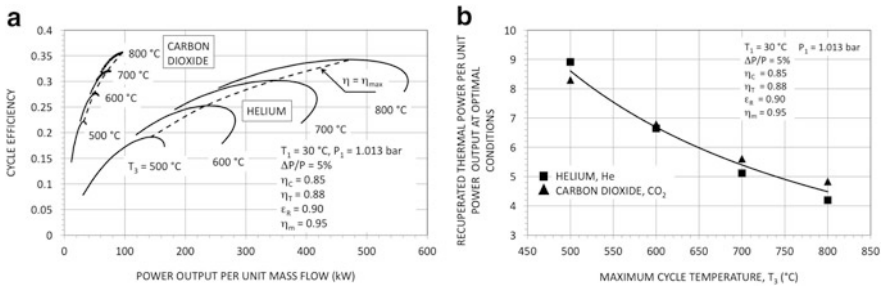
For this reason, Fig. 1.38d represents two cycles, on the thermodynamic plane T-S, with the same operating conditions, but two different working fluids: helium and carbon tetrafluoride. The maximum temperature  $T_3$  is assumed to be  $500^\circ\text{C}$  for both fluids (probably the limit value of thermal stability of CF<sub>4</sub>); the minimum temperature  $T_1$  is  $30^\circ\text{C}$ . The minimum pressure of the cycle  $P_1$  is the same as the atmospheric pressure. The optimal efficiency is reached in both cases when  $\psi = 0.27$  (with their very different compression ratios  $r_c$ : 1.7 for helium, 5 for carbon tetrafluoride) but the efficiencies are, in these hypotheses, about 0.19 for the helium cycle and 0.23 for the CF<sub>4</sub> cycle.

There is an analysis of the distribution of efficiency losses with respect to the ideal value  $\eta_{\max} = 1 - T_1/T_3$  (see (1.22) and Appendix C) in Fig. 1.39. As a comparison, the figure shows the values of cycle efficiency losses  $\Delta\eta_j$  for a cycle with carbon dioxide, too. The thermodynamic loss introduced by the pressure losses ( $\Delta\eta_2$ ) is that which, passing from the case of the helium cycle to the CF<sub>4</sub> cycle, is subject to the greatest change: from 0.043 efficiency to 0.012 (a reduction of 72 %). The increase in the loss on the regenerator ( $\Delta\eta_3$ ) does not compensate for the dramatic reduction in loss related to the pressure losses, and the efficiency of the CF<sub>4</sub> cycle is around 22 % higher than the efficiency of the helium cycle. In principle, this gives an advantage at the design stage. The reason for the reduced impact of the pressure losses in a cycle that uses a gas with high molecular complexity can be shown by estimating the efficiency loss  $\Delta\eta_2$  in the case of a perfect gas:





**Fig. 1.39** Efficiency losses for gas cycles with optimal operating conditions, using different working fluids



**Fig. 1.40** (a) Cycle efficiency as a function of the useful power per unit flow at various maximum temperatures of the cycle. (b) Ratio between the thermal power recuperated at the regenerator and the useful power under conditions of maximum cycle efficiency, with variation in the maximum temperature

$$\Delta\eta_2 = \frac{T_1}{\dot{Q}_{in}} \Delta\dot{S}_{G,2} \approx \frac{T_1}{C_p(T_3 - T_6)} \left( -R \ln \frac{P_{2a}}{P_2} \right) = -\frac{T_1}{T_3 - T_6} \frac{\gamma - 1}{\gamma} \ln \left( 1 - \frac{\Delta P}{P_2} \frac{1}{100} \right)$$

Since, generally speaking, the temperature difference  $(T_3 - T_6)$  is constant (having fixed the  $\psi$  ratio and the machinery efficiency), having also established the ratio  $\Delta P/P_2$ , we can see how the gases with modest  $\gamma$  values (high molecular complexity) have  $\Delta\eta_2$  values that are inferior to the corresponding values for gases with higher  $\gamma$  values.

The area enclosed by the cycles in Fig. 1.38d is approximately proportional to the useful work. In fact, although  $W/(C_p T_1)$  for the CF<sub>4</sub> cycle is higher than that of the helium cycle (see Fig. 1.38c), the power per unit mass flow is about 30 kW and 140 kW, respectively, for CF<sub>4</sub> and helium.

Figure 1.40a reports cycle efficiencies and the useful power per unit gas mass flow for helium cycles and carbon dioxide cycles. The maximum temperature  $T_3$

varies from 500 °C to 800 °C. Figure 1.40b reports the thermal power recuperated per unit of useful power, for the optimal values of cycle efficiency.

With  $T_3$  equal to 800 °C, maximum cycle efficiencies are similar (0.34 and 0.36, respectively, for helium and CO<sub>2</sub>); the corresponding compression ratios are 2 for the helium cycle and 4 for the CO<sub>2</sub> cycle. The useful power per unit gas mass flow for the helium is five times greater than that for the CO<sub>2</sub> cycle.

The thermal power on the regenerator is invariably much greater than the useful power (see Fig. 1.40b): around 4.5 times, at  $T_3$  of 800 °C. It grows rapidly as the maximum temperature falls (around 8.6 times at  $T_3$  of 500 °C). As a result, the regenerator is a very difficult and delicate component to design: the thermal power that it needs to operate with is very big (compared to the useful power), and its efficiency needs to be as high as possible if good thermodynamic performance is to be guaranteed (see also Appendix C.)

### ***1.7.6 Final Considerations on the Choice of Working Fluid and Pressure Levels in the Gas Cycle***

Practical considerations such as thermal stability, chemical inertia, toxicity, and nonflammability eliminate many potential candidates among the organic fluids, and in fact, there are only a few inorganic fluids that have been considered for use in the closed Brayton cycles.

In the plants built to date (see Sect. 1.7.7), air has been the most commonly used working fluid. When planning engines for space power systems, optimisation of weight and size has generally restricted the choice of working fluid to the monoatomic gases with an average molar mass: neon, argon and mixtures of helium and xenon.

In the first generation of gas nuclear reactors, carbon dioxide and helium were used just for cooling the reactor and for generating steam in the steam generators. The idea of using direct cycles with helium has been considered on several occasions.

Previous analysis showed that the choice of fluid to be used in a closed gas cycle is made difficult by the opposing needs to be met when planning the size of the turbomachinery and the heat exchangers: if in order to minimise the number of stages in a turbomachine, a gas with a low specific heat (specific to the mass) is preferred, on the other hand, in order to minimise the mechanical power needed to circulate the gas in the heat exchangers, we need a gas with a high density and specific heat. Since the density may be manipulated to a suitable value by changing the pressure in the plant, the problem can be limited to a discussion on the role of specific heat. To summarise the above, considering the influence of the molecular structure and its molar mass separately:

The structure of the molecule could be:

- Simple (high value of  $\gamma = C_p/C_v$ ): this is always preferred, since it makes it possible to operate with low optimised compression ratios. This represents an advantage for the turbomachinery, because it permits a moderate variation in the flow volumes and average diameters along the machine shaft and, consequently, in the typical number of revs, with the benefit of high efficiency in the turbomachinery. As far as the exchangers are concerned, a low compression ratio means minimum and maximum pressure values in the cycle have the same order of magnitude: this makes it possible to maintain the right pressure in the low-pressure side, too, and, therefore, not to penalise the exchangers in charge of the heat release (regenerator and secondary exchangers; see (1.41) and (1.42)).
- Complex (low value of parameter  $\gamma$ ): gives a high optimised compression ratio and, consequently, a tendency towards poor efficiency in the turbomachinery (both for the significant variations in the volume flows and for the potentially supersonic speeds in the turbine) and the need for great mechanical power to circulate the gas in the heat exchangers at low pressure (for the combined effect of the specific heat and the density).

Therefore, it makes sense to restrict the analysis to the simple molecule gases, which appear preferable from both points of view. We still need to examine the effect of the molar mass of the gas, which may be:

- Low (e.g. helium) which has a relatively high specific heat, desirable for heat exchangers, but, although not affecting a priori their efficiency, does lead to a large number of stages in the turbomachinery.
- High (e.g. argon, xenon) which is favourable for the size of the turbomachines (small number of stages) but requires the use of very large heat exchangers.

The choice between these two opposites will be of an economic nature and depend on the size of the plant: if the plant needs great power, then the impact of the cost of the exchangers on the total plant costs will be significant and, therefore, the prevalent choice will be for helium. If the plant is on a small scale, then the heat exchangers are relatively standard components and not very expensive, but the cost of the turbo-compressor group becomes fundamental: in this case, a gas with a high molar mass is favoured. There are certain plants (e.g. in space applications; see [29]) in which the economic optimisation has led to the use of a mixture of helium and xenon, mixed in such a way as to obtain a gas with a preset molar mass.

As the pressure losses are always critical in the closed gas cycles, in principle, preference is always for a working fluid with high exchange heat transfer coefficients and modest pressure losses.

Various closed-cycle plants have used air (see Sect. 1.7.7), but the presence of oxygen in high quantities leads to corrosion and limits its use to relatively low temperatures [30]. Carbon dioxide has been employed in several gas-cooled nuclear reactors, but it has a high propensity to oxidation with common steels. Argon is not desirable on account of its limited heat transfer qualities (see Sect. 1.7.4), but it is widely available (it is around 1 % by volume in air) and could be used in small power

engines. Helium is inert, but it is difficult to obtain with a high degree of purity. It may contain significant quantities of hydrogen (about 5 ppm by mass), water vapour (about 50 ppm) and air (about 75 ppm). These impurities may accelerate the phenomena of oxidation and creep [31].

In a closed gas cycle, pressurisation plays an important role in the size of the cycle components. For a rough comparison in the size of the components, let's assume (1) the temperatures and the speeds of the gas are fixed at the various points of the cycle and (2) the effects of the Reynolds number are negligible, with  $P_0$  being the low-pressure reference value and  $P$  the pressure, higher than  $P_0$ , at which the system is pressurised. This means that at equal mass flows (or equal useful power)

$$\frac{D}{D_0} = \sqrt{\frac{P_0}{P}}$$

the typical diameters of the various plant components are reduced as the pressure is increased. In the specific case of the turbomachinery, the consequence is an increase in the number of revs

$$\frac{N}{N_0} = \frac{D_0}{D} = \sqrt{\frac{P}{P_0}}$$

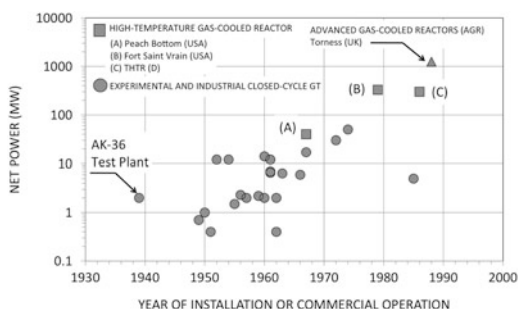
As a result of reducing the diameters, we can design closed gas cycles with high power using smaller machinery than that in the open gas cycles. Or, with machinery of the same size, we can obtain more useful power.

As far as the heat exchangers are concerned, in particular the recuperator, see Sect. 1.7.4, generally speaking, once the geometry of the recuperator has been fixed, the pumping power is inversely proportional to the square of the fluid density (or the pressure).

Increasing the pressure also has a positive effect on the internal efficiency of the turbomachines, and this tends to increase as the Reynolds number rises [32].

The pressurisation may also be exploited to regulate the power output. In fact, if we suppose that the machine supplies the nominal power and we wish to reduce it by 50 %, since in order to reduce the power, we need only half the flow of fluid circulating in the plant, leaving the useful work unchanged (which usually corresponds to the optimised solution), it is enough to halve the maximum pressure of the cycle. In this way, we halve the density of the fluid and, consequently, the circulating mass flow. Leaving the temperature extremes of the cycle untouched, there is no significant change to the efficiency. From the point of view of the plant, in order to carry out the regulation, there needs to be a tank to which the necessary quantity of gas can be added or subtracted; furthermore, it should be emphasised that, during operation with a partial load, the heat exchangers will be super-sized and can therefore compensate for the fall in the exchange coefficient caused by the reduced pressure, thereby leaving the points of the cycle largely unchanged.

**Fig. 1.41** Plants with closed gas turbines and gas-cooled reactors built up to 1990



### 1.7.7 Examples of Several Plants that Have Been Built and Examples of the Engines Proposed

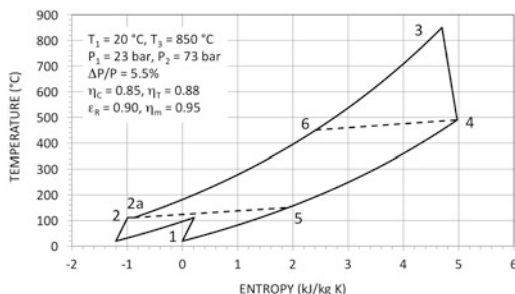
In the industrial gas-turbine plants with closed cycles that were built between the 1950s and 1970s, the heat, deriving from combustion outside the engine, placed a limit on the temperature of the exchangers and, consequently, on the maximum temperature of the thermodynamic cycle. The compression has always been intercooled in order to reduce the power absorbed. Between 1939 and 1972, 23 experimental prototypes and industrial machines were built based on a closed-cycle gas turbine, with power supplies varying from 0.4 to 50 MW ([33] and see Fig. 1.41).

In most cases, the working fluid used was hot air. The plant at Paris in 1952, with 12 MW, was probably the most complicated turbine of this kind that was made: with three intercoolers for the compressor, a recuperator and an intermediate expansion heating. The machine was built with two shafts. The oil-operated combustion chamber was turbocharged, so that smaller heating surface areas were needed. The pipes were in austenitic steel Cr/Ni 18/8. Conditions at the input of the HP turbine were 658 °C and 54 bar; those at the input of the LP turbine were 686 °C and 20 bar. The minimum cycle pressure was 5 bar. The high-pressure fast shaft turned at 8,000 rpm, and the low-pressure one was synchronised at 3,000 rpm. The net efficiency was 28.23 %.

Another important plant was built at Ravensburg, in 1956, with 2.3 MW. This plant operated for 120,000 h without any particular problems. Its efficiency was 25 % with fuel oil and 23 % with coal. The minimum pressure was 8 bar and the maximum 30 bar. The maximum temperature of the cycle was 660 °C.

The La Fleur engine, a helium turbine from 1962, developed by James La Fleur of Los Angeles was probably the first helium turbine built. The engine was developed for an air liquefaction and separation process, and the turbine, interlocked with an inverse Brayton cycle, produced no useful electrical energy. The minimum pressure in the system was 12.7 bar, with the maximum 18 bar, and a temperature at the start of expansion of 650 °C. A second helium plant of 50 MW was built in 1974: the Oberhausen II helium turbine.

**Fig. 1.42** Helium cycle for HTR (High Temperature Reactor) power plants



Various nuclear plants, some with high power supplies (up to 1,000 MW) were built between 1970 and 1990 (see Fig. 1.41). The gas (carbon dioxide in the advanced gas-cooled reactor (AGR) and helium in the high temperature gas-cooled reactor (HTGR) reactors) was used to cool the reactor and then to generate steam for the power-generating section by means of a heat exchanger (the steam generator).

In 1985, the Garrett Corporation (USA) made a closed gas cycle, with air as the working fluid, associated with an atmospheric fluidised bed combustor burning a low-grade fuel. The temperature at the input of the turbine was  $790\text{ }^{\circ}\text{C}$  and the plant worked apparently perfectly well, but without ever being commercialised [34].

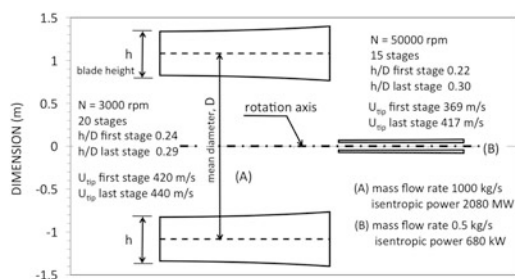
## Exercises

**1.8.** Figure 1.42 shows a high power helium cycle in the thermodynamic plane T-S (net power of about 1,000 MW), designed for high-temperature nuclear plants [33, p. 183]. In this example, the maximum temperature is  $850\text{ }^{\circ}\text{C}$  and the minimum has been assumed to be  $20\text{ }^{\circ}\text{C}$ . With the assumed parameters, the cycle efficiency is around 40%.

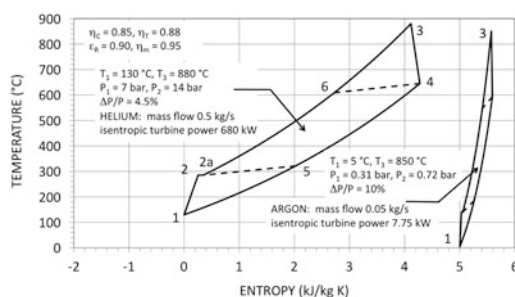
The thermal power recuperated in the regenerator is 2.14 MW for every MW of useful power. The intercooled compression significantly reduces the compression work, increasing the useful work, to the benefit of efficiency. The total compression ratio is divided into two equal parts between the compression groups.

The geometrical dimensions of the turbine have been summarised in Fig. 1.43, case (A). There are many stages (20 axial stages) to prevent excessive peripheral speeds and to limit the centrifugal mechanical stress on the rotor blades. The number of revs is synchronous (3,000 rpm). The specific number  $\omega_s$  is 0.98 for the first stage and 1.28 for the last stage. The ratio height of blade/average diameter  $h/D$  is 0.24 for the first stage and 0.29 for the last stage.

The great amount of useful power justifies the costs and the complexity of the intercooling as well as the use of helium and the demanding turbo-compression group that it entails.



**Fig. 1.43** Qualitative dimensions for turbines in helium cycles. (A) Turbine for the thermodynamic cycle of Fig. 1.42, with an isentropic power of 2,080 MW. (B) Turbine for the thermodynamic helium cycle of Fig. 1.44, isentropic power of 680 kW.  $h$  represents the blade height;  $D$  is the mean diameter

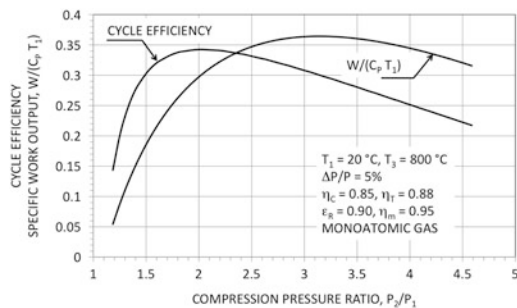


**Fig. 1.44** Two thermodynamic gas cycles in closed cycles with moderate power and with different working fluids. Helium, isentropic expansion power of 680 kW. Argon, isentropic expansion power of 7.75 kW

**1.9.** Figure 1.44 represents, on the plane T-S, two low-powered cycles: one cycle with helium and the other with argon. The helium cycle [29], having set the power (about 200 kW), has a mass flow capacity of 0.5 kg/s but an elevated enthalpy isentropic drop (about 1,440 kJ/kg). The consequence is a small turbine (with a mean stage radius of about 6 cm; see Fig. 1.43, case (B)), with 50,000 rpm and 15 stages.

The argon cycle (useful power 3 kW [35]) is completely sub-atmospheric. The flow, which is ten times inferior to that of the helium cycle above, combined with a molar mass of the fluid around ten times greater, means that the power of this cycle is one hundredth that of the helium one. The turbine, single-stage axial with total admission, at 65,000 rpm, has an average radius of 5.8 cm and a  $h/D$  ratio (height of blade compared to average diameter of the rotor) of 0.058. The degree of reaction at the mean diameter is assumed to be 0.5.

**1.10.** For a closed gas cycle, assuming the parameters reported in Fig. 1.45, we find that the maximum efficiency (see the curve “cycle efficiency” of Fig. 1.45)



**Fig. 1.45** Trends of the cycle efficiency and of the specific work output  $W/(C_p T_1)$  as a function of the compression pressure ratio for monoatomic gases

**Table 1.6** Several results for two closed gas cycles under conditions of maximum efficiency

|                                                                  | Helium  | Argon  |
|------------------------------------------------------------------|---------|--------|
| Expansion work (kJ/kg)                                           | 1,333.1 | 133.58 |
| Compression work (kJ/kg)                                         | 702.73  | 70.388 |
| Recuperated thermal energy (kJ/kg)                               | 2,117.2 | 212.18 |
| Recuperated energy/network                                       | 3.756   | 3.755  |
| Cycle efficiency                                                 | 0.359   | 0.359  |
| Mass flow (kg/s)                                                 | 0.0214  | 0.2128 |
| Total $UA$ of the recuperator ( $W/^{\circ}\text{C}$ )           | 1,000.0 | 990.77 |
| Total volume $V$ of the recuperator ( $\text{cm}^3$ )            | 1,067   | 10,489 |
| Equivalent length of the side of a cube with the volume $V$ (cm) | 10.22   | 21.89  |

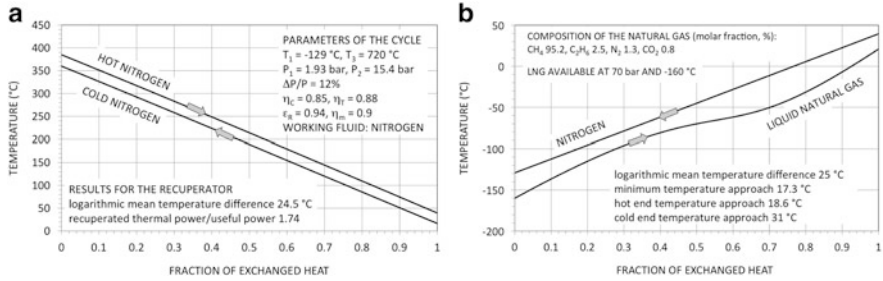
The useful power is 10 kW for both cases. The data and parameters that have been assumed for our calculations are to be found in Fig. 1.45

corresponds to a compression ratio  $P_2/P_1 = 2$ . With reference to helium and argon (two monoatomic gases), we compare and discuss the dimensions of the recuperator. We assume a useful power of 10 kW. Some results of the cycle calculation are provided in Table 1.6.

Without presuming to make detailed calculations about the regenerator, we shall limit ourselves to calculating the total volume necessary. The choice of geometry for the exchanger has been made without regard for the pressure losses. In fact, with adequate pressurisation these can be reduced at will.

Choosing the surfaces, the flow areas in the two branches, plus the need to keep pressure losses to a minimum (basically, higher on the side with the lower density fluid) and the actual form of the exchanger make a complete calculation of the regenerator fairly complicated. The complete optimisation of the regenerator (the calculation of the volume necessary for the heat transfer and the evaluation of the pressure losses) should not disregard the needs of the turbo-compressor group. At this time, though, we shall limit ourselves to a preliminary calculation of just the volume required to guarantee the exchange of the necessary thermal power.





**Fig. 1.46** Exercise 1.11. (a) Diagram of heat transfer for the recuperator in a gas cycle. (b) Diagram of heat transfer in the exchanger used for the regasification of LNG

The surface in question is the plain plate-fin surface #46.45T, the geometric characteristics of which, as well as the heat transfer, are reported in [26, p. 234]: the same is true for both sides of the exchanger. We assume a cross-flow configuration for the exchanger. The  $UA$  product of the global coefficient of heat transfer  $U$  and of the area  $A$  necessary for the heat transfer is known from the cycle calculations, having set the efficiency  $\epsilon_R$  of the regenerator. The gas mass flow  $\dot{m}$  is derived from the power-required desiderata (10 kW). Having chosen the surface type for the heat transfer, the main geometrical measurements are known, and it is possible to calculate for both flows (1) the coefficient  $\alpha = A/V$ , ratio of the total heat transfer area on one side to the total volume of the exchanger, and (2) the coefficient  $\sigma = A_c/A_{\text{fr}}$ , the ratio of the free-flow area to the frontal area. In our case, the total heat transfer areas are the same for both the hot and cold side.

Therefore, for both flows (that on the cold side and that on the hot side), having set a value for the flow-stream mass velocity  $G$ , it is possible to calculate the free-flow area  $A_c = \dot{m}/G$  and then the frontal area  $A_{\text{fr}} = A_c/\sigma$ . The temperatures of the gas give us all the interesting physical properties of the gas, and we can evaluate the Reynolds number  $Re$ , the Stanton number  $St$  (for the surface type chosen) and the heat transfer coefficients  $h$ .

Having calculated the overall coefficient of heat transfer  $U$  (referring to the area on one of the two sides of the exchanger), from the product  $UA$ , we derive the area  $A$  (which, in our case, is the same for both sides). The volume  $V$  is found, at this point, as  $V = A/\alpha$ . The result is in Table 1.6: in the case of the argon cycle, the volume is ten times greater than that necessary for the helium cycle.

**1.11.** As a final example of a possible application of a gas turbine in a closed cycle, we consider a nitrogen cycle (see [33]) designed for the regasification of liquid natural gas (LNG, Liquefied Natural Gas). Design data for the thermodynamic cycle are reported in Fig. 1.46a, with the relative diagram of the exchanged heat in the recuperator. Assuming these parameters, the cycle efficiency is 51.8%. The waste heat, available (see Fig. 1.46b) between the temperatures  $T_5$  and  $T_1$  of  $39.55^\circ\text{C}$  and  $-129^\circ\text{C}$ , heats a flow of LNG in supercritical conditions (available at 70 bar and  $-160^\circ\text{C}$ ) up to about  $20^\circ\text{C}$ . The logarithmic mean temperature difference assumed in the exchanger that regases the LNG is  $25^\circ\text{C}$ , but, on account

of the variation in the heat capacity of the LNG during heating, the differences in temperature between the two flows (the hot one consisting of nitrogen and the cold one consisting of LNG) vary from a minimum of 17.3 °C to a maximum of 39.2 °C. The hot and cold end temperature approach differences, result 18.6 °C and 31 °C, respectively.

A parameter of merit for a cycle designed for the regasification of natural gas is the net (electrical) power generated per unit flow of LNG, expressed in MW/(kg/s): the Specific Power Performance Parameter,  $SPP = \dot{W} / \dot{m}_{LNG} = \Delta H_{NLG} \times \eta / (1 - \eta)$ . For this example, we have  $SPP = 0.826 \text{ MW}/(\text{kg/s})$ .

## 1.8 The Stirling Engine

The Stirling engine forms part of the family of “air engines” or “hot-air engines”. Inside it, a mass of gas (air, in the earliest engines; nowadays, nitrogen, helium or hydrogen at high pressure) expands and contracts in a cycle, as it is heated then cooled, producing mechanical work.

The Stirling engine incorporates a regenerator which functions as a thermodynamic sponge, alternately absorbing and releasing heat to the gas which passes through it. It is the presence of this regenerator which, theoretically, enables the Stirling engine to reach its maximum efficiency, once the two limit temperatures  $T_H$  and  $T_0$  have been set (see Sects. 1.1 and 1.5, Fig. 1.11). The engine operates in a closed cycle.

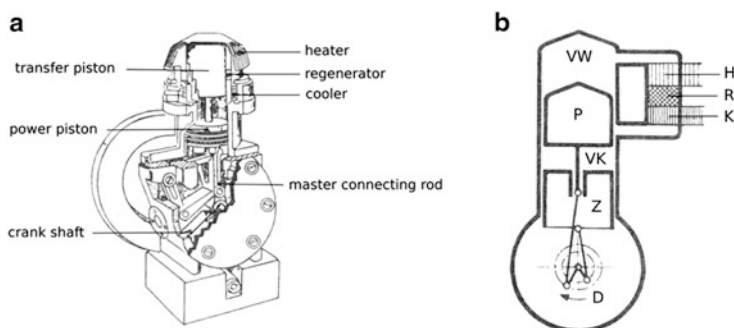
Reverend Robert Stirling<sup>29</sup> invented the regenerator and the new engine that used it in 1816: this was the regenerative air engine, which would subsequently bear his name. Rev. Stirling and his brother James<sup>30</sup> constructed several engines over the following years. Two years after the patent, the first engine was built for pumping water from a quarry in Ayrshire (Scotland).

The air engines were used as an alternative to steam engines, towards the end of the 1800s. However, the rapid development of the internal combustion engines (Otto and Diesel) led to their decline. It was only towards the end of the 1930s that the Philips Electric Company, Eindhoven, started to study them once more and created several prototypes with good efficiency and small power supply, but without any real commercial success. For example, the engine described in [36] has a power of 180 W with an engine speed of 1,500 rpm (a diagram of the engine is shown in Fig. 1.47). General Motors, under licence from Philips, subsequently built engines with power ranging between 7–8 kW and 600 kW (for locomotives and ships). That was until 1970. Then the Ford Motor Company, again under licence from Philips, took over from General Motors, but research was called off in the early 1980s.

However, the studies into the engine were never wholly abandoned: in the 1960s, William T. Beale made a significant contribution to the development of

<sup>29</sup>Robert Stirling (1790–1878), a Minister in the Church of Scotland at Galston, Ayrshire.

<sup>30</sup>James Stirling (1800–1876), a Scottish engineer.



**Fig. 1.47** (a) Drawing of the Philips air-engine generator MP 1002 CA. (b) Simplified diagram of the air engine. The heater (H), the regenerator (R), and the cooler (K) are shown, for clarity's sake, at one side of the cylinder and not around it (From Author [36])

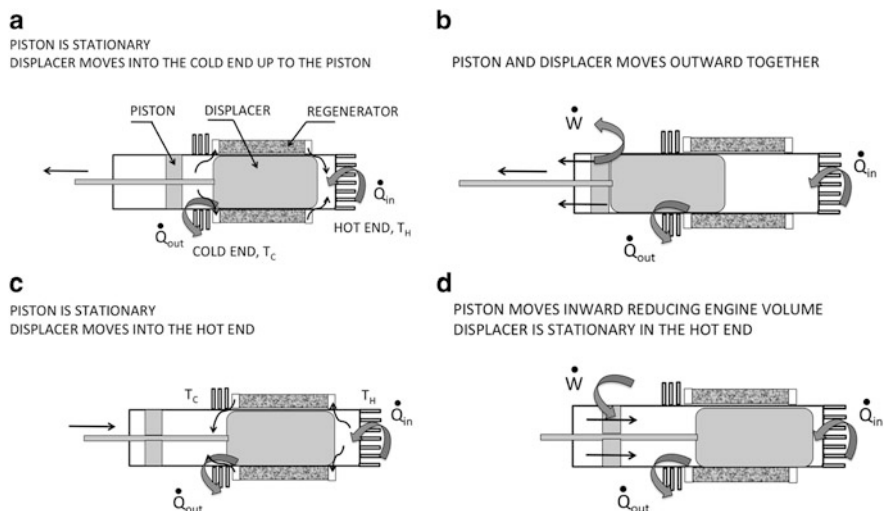
the free-piston version (the free-piston Stirling engine, in which the movement of the internal parts of the engine is guaranteed by dynamic actions, without having to resort to kinematic couplings<sup>31</sup>); from 1964 until the 1990s, research into the Stirling engine focussed on the possibility of using it to make an artificial heart [37]; towards the end of the 1970s and up to 1990, various underwater propulsion units for non-nuclear propulsion submarines which did not require air as a comburent were designed, mainly with the contribution of the German group MAN–MWM and the Swedish company Kockums (which still has some of these engines in production) [38]. Today, there are numerous companies offering Stirling engines, above all for micro-cogeneration, often associated with the combustion (or gasification) of biomass, with electrical power of up to several dozen kW.

In fact, any Stirling engine can also operate as a refrigerator: supplying mechanical energy and removing the heat source at high temperatures. In this way, the volume designated for the expansion absorbs heat at low temperature, generating a cooling effect. By using cryogenic fluids (e.g. nitrogen, hydrogen, helium) as working fluids, we create the so-called miniature cryocoolers (with power levels of even less than 1 W). These will not be discussed here, but this is a sector in which the Stirling machine is widely used these days (although the market is relatively small).

The ideal thermodynamic cycle by which the engine operates is similar to that shown in Fig. 1.11 (the Ericsson cycle<sup>32</sup>) but with two isocores in place of the two isobars. An indication of how the engine works can be seen in Fig. 1.48. The basic components of the engine are as follows: a piston, which enables the exchanges of mechanical power; the “displacer”, which transfers the gas in alternating fashion

<sup>31</sup> Another peculiarity of these engines is their self-starting capacity. By heating the expansion space and cooling the compression volume, the engine starts up automatically.

<sup>32</sup> From John Ericsson (1803–1889), an engineer born in Sweden. He emigrated to America and created numerous technically different engines.



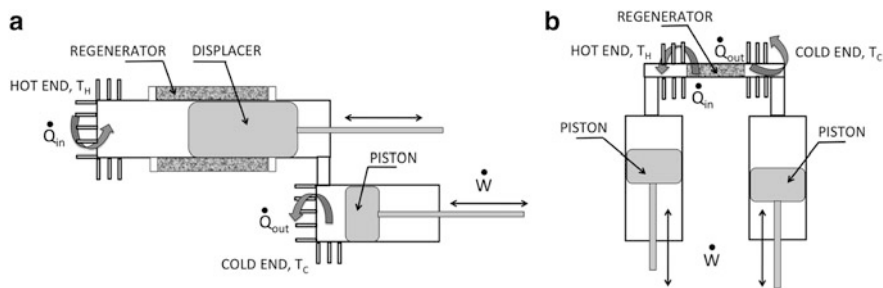
**Fig. 1.48** The four phases of the Stirling engine with piston and displacer in the same cylinder (“beta” configuration). (a) Mass transfer stroke (from the cold to the hot side). (b) Expansion stroke. (c) Mass transfer stroke (from the hot side to the cold). (d) Compression stroke

from the cold to the hot head of the engine; and the regenerator, which, during the transfer of the gas from the hot volume to the cold, absorbs heat from the gas or releases heat to the gas passing through it. The presence of the regenerator is necessary if we want the engine to operate at high efficiency (although it does introduce significant pressure losses which, if they are excessive, could cancel out the useful work). The regenerator may be either external to the cylinder that contains the piston-displacer system, inside the displacer or absent. In the latter case, it is the annulus between the displacer and the cylinder that operates in place of the regenerator (by means of the cylinder and displacer walls) and it is known as regenerative annulus.

In the situation shown in Fig. 1.48a, the piston is stationary at its upper dead point, and the displacer transfers the gas from the cold head to the hot one. During the transfer (which actually happens without consuming work and, in the ideal case, with a constant total volume), the gas is forced to pass through the regenerator, which preheats it before it is entirely displaced from the cold space to the hot one.

The pressure of the gas in the hot space increases, as a consequence of the thermal power  $\dot{Q}_{in}$  that is introduced. The piston-displacer system begins to move, supplying mechanical power  $\dot{W}$  (Fig. 1.48b), reaching the lower dead point, corresponding to where the pressure will be at its lowest designed value.

When the piston reaches the end of its stroke (Fig. 1.48c), the displacer transfers the hot gas towards the cold space, and during this transfer, the gas, as it cools, releases part of its internal energy to the regenerator. When all the gas reaches the cold end of the engine, the piston presses it, increasing its pressure (Fig. 1.48d) and reaching once more the new upper dead point. At the same time as the compression,



**Fig. 1.49** Two further configurations of the Stirling engine. **(a)** Piston and displacer in separate cylinders (“gamma” configuration). **(b)** Two piston machine (“alpha” configuration)

there is also an extraction of thermal power  $\dot{Q}_{\text{out}}$  from the engine. At this point, the thermodynamic cycle can start again. Unlike the closed cycles with external combustion based on the Rankine or Brayton cycles, the working fluid in the Stirling engines does not follow a unidirectional flow, but oscillates continuously inside the volume that constitutes the engine.

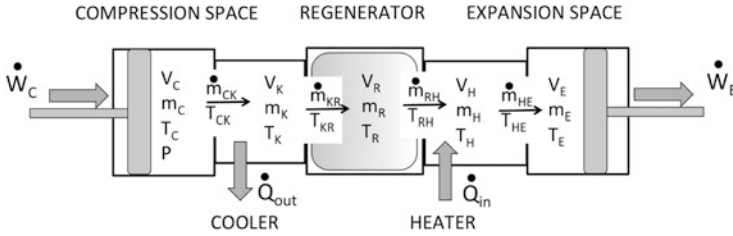
Another two classic types of engine are shown in Fig. 1.49. A great variety of mechanisms have been developed for connecting the various pistons. Several configurations of this engine, as already mentioned, have pistons that are not mechanically coupled (free pistons). Other layouts (with a hybrid coupling) have the power piston coupled mechanically with the exterior and the displacer free. A typical engine of this type is the Ringbom<sup>33</sup> machine [39].

The real thermodynamic cycle by which the working fluid inside the engine operates does not follow the ideal cycle at all: the pistons and the displacer do not have a discontinuous movement; the gas is not confined each time to just the well-defined volumes of expansion and compression, but distributed throughout the whole area of the engine; the expansion and the compression are not isothermal; the cooling and heating sections, which are physically distinct from the volumes of expansion and compression, introduce dead spaces<sup>34</sup>; the regenerator, albeit fundamental, introduces a further dead space; the regeneration is not perfect. Then there are the losses due to mechanical friction and the windage effects. The main losses inside the engine are thermal in nature, due to heat conduction from the hot expansion region to the cold compression space and dynamic (essentially the pressure losses, which depend on the number of revs and on the mean operating pressure).

The spaces in which the compression and expansion take place vary in time cyclically and simultaneously, but not in phase. They are connected by the regenerator

<sup>33</sup>Ossian Ringbom, Finnish, “subject of the Czar of Russia, residing at Borga” patented in the United States (Patent no. 856102, 4 June 1907. Application filed 17 July 1905) and in the UK (Patent no. 10675, 22 May 1906. Date of Application, 22 May 1905) his own hot-air engine.

<sup>34</sup>The dead spaces reduce the compression ratio and the specific useful power.



**Fig. 1.50** The ideal adiabatic reference model with five volumes

and two auxiliary exchangers: the cooler and the heater. The general conceptual layout, which is valid for all the “alpha”, “beta” or “gamma” configurations [40, p. 91], is shown in Fig. 1.50, with the two opposed pistons. We shall refer to it for the mono-dimensional thermodynamic analysis of the engine.

### 1.8.1 The Thermodynamic Analysis of the Stirling Engine

The first thermodynamic analysis of the Stirling cycle was carried out by Gustav Schmidt, a professor at the German Polytechnic Institute in Prague, in 1871 (55 years after Robert Stirling was granted his patent for the engine in 1816). Although Schmidt’s analysis refers to an ideal machine, with isothermal compression and expansion, it has often been used since then as the reference for a preliminary analysis of the characteristics of any Stirling engine. Not till 1960 did Theodor Finkelstein [41], incorporate a more realistic hypothesis for adiabatic compression and expansion into the analysis, considering the spaces of expansion and compression as physically separate from those of cooling and heating. Even considering ideal exchangers (absence of pressure losses and infinite coefficients of heat exchange), Finkelstein’s analysis still gives slightly more realistic results than Schmidt’s analysis.<sup>35</sup>

Hereafter, we will refer exclusively to the adiabatic analysis; Schmidt’s theory, with the perfect gas working fluid, is described and discussed in numerous textbooks, for example, in [42, 43]. The adiabatic analysis, although providing results far different from effective reality, is still useful for our purposes. It highlights the basic thermodynamic aspects linked to the choice of working fluid and the operating conditions of the cycle.

<sup>35</sup>The engine efficiency calculated by applying Schmidt’s analysis (the ideal isothermal analysis), for example, coincides with that of the ideal thermodynamic cycle (the efficiency of Carnot’s machine), whilst the hypothesis of adiabaticity (the ideal adiabatic analysis) leads to lower efficiency, around 20–30 % lower than that of the ideal isothermal model. However, this is still significantly higher than that of a real engine.

As mentioned (see Sect. 1.8), there are many mechanical configurations for the Stirling engine, but, in each case, the basic parameters behind the engine design are (referring to Fig. 1.50):

- The minimum temperature  $T_K$  at the cooler and maximum temperature  $T_H$  at the heater. In other words, the temperature ratio  $\tau = T_H/T_K$ .
- The swept compression volume  $V_{SW,C}$  and the swept expansion volume  $V_{SW,E}$ . The corresponding swept volume ratio may be defined as  $\kappa = V_{SW,C}/V_{SW,E}$ .
- The total dead volume  $V_D$ , defined by means of the dead volume ratio  $\xi = V_D/V_{SW,E}$ .
- The kinematic phase angle  $\phi$  between the two pistons (see relation (1.43b)). The kinematic phase angle introduces consequently a lag between the laws of variation over time of the volumes of expansion and compression.
- The speed of the engine  $N$ .
- The pressure  $P$ : instantaneous, mean, maximum, and minimum.

In Fig. 1.50, the engine is drawn in five different volumes:

- The compression volume  $V_C$  which contains the gas at temperature  $T_C$
- The volume of the cooler  $V_K$
- The volume of the regenerator  $V_R$
- The heater volume  $V_H$
- The expansion volume  $V_E$  which contains the gas at temperature  $T_E$

The two pistons that control the variations in the compression and expansion volumes are connected to the same shaft; the cooler, the regenerator and the heater are interposed between the two pistons. The two temperatures  $T_C$  and  $T_E$ , assuming that the two volumes  $V_C$  and  $V_E$  are adiabatic, change over time. On the other hand, the temperatures  $T_K$  and  $T_H$  are assumed to be constant and equal to the respective temperatures of the gas enclosed inside volumes  $V_K$  and  $V_H$ .

Assuming that the movement of the pistons is sinusoidal, the compression volume  $V_C$  and the expansion volume  $V_E$  vary over time, in accordance with equations [43]

$$V_C = V_{CL,C} + \frac{1}{2} V_{SW,C} [1.0 + \cos \omega t] \quad (1.43a)$$

$$V_E = V_{CL,E} + \frac{1}{2} V_{SW,E} [1.0 + \cos (\omega t + \phi)] \quad (1.43b)$$

in which  $\omega = 2\pi N/60$  is the angular velocity;  $V_{CL,C}$  and  $V_{CL,E}$  are, respectively, the clearance spaces in the compression and expansion volumes; and  $V_{SW,C}$  and  $V_{SW,E}$  are the two swept volumes. In the examples below, we shall assume that (1.43a) and (1.43b) are always valid.

The total volume of the engine  $V_T$  will therefore be equal to

$$V_T = V_{CL,C} + V_{SW,C} + V_K + V_R + V_H + V_{CL,E} + V_{SW,E} \quad (1.44)$$

whilst the dead volume  $V_D$  is

$$V_D = V_{CL,C} + V_K + V_R + V_H + V_{CL,E} \quad (1.45)$$

The quantities variable over time to be calculated, in a time interval equal to the period, are:

- The pressure  $P$  in all the volumes that the engine has been drawn with
- The temperature  $T_C$  of the gas in the compression volume
- The temperature  $T_E$  of the gas in the expansion volume

Knowing  $P$ ,  $T_E$ , and  $T_C$ , we can calculate the gas masses  $M_C$ ,  $M_K$ ,  $M_R$ ,  $M_H$ , and  $M_E$  contained in the various volumes and then:

- The gas mass flow  $\dot{m}_{CK}$  of the compressor to the cooler (negative if the movement is from the cooler to the compressor)
- The gas mass flow  $\dot{m}_{KR}$  from the cooler to the regenerator (negative if the movement is from the regenerator to the cooler)
- The gas mass flow  $\dot{m}_{RH}$  from the regenerator to the heater (negative if the movement is from the heater to the regenerator)
- The gas mass flow  $\dot{m}_{HE}$  from the heater to the expansion volume (negative if the movement is in the opposite direction)

Another two parameters that need calculating are the temperature  $T_{RK}$  of the gas when it flows from the regenerator to the cooler and the temperature  $T_{RH}$  of the gas when it passes from the regenerator to the heater.

If there are no pressure losses on the heat exchangers and the regenerator, the pressure  $P$  is the same in all the volumes; if the working gas is a perfect gas and the regenerator is ideal, then  $T_{RK} = T_K = T_{KR}$  and  $T_{RH} = T_H = T_{HR}$ .

Therefore, the useful work in the cycle is  $W = | \dot{W}_E | - | \dot{W}_C | = \oint (| \dot{W}_E | - | \dot{W}_C |) dt$ , and the cycle efficiency can be calculated as  $\eta = W / \oint \dot{Q}_H dt = W / Q_H$ .

The calculation model in the following described and used is not intended to be an instrument for making a detailed design of an engine. Its main purpose is to facilitate discussion about the thermodynamic effects and to emphasise (if only in a qualitative way) the effect of fluid dynamic losses on performance (power and efficiency) of the cycle, by means of a few global parameters.

### *The Ideal Engine*

In the ideal case, the balance of energy applied to the compression and expansion volumes, ignoring the kinetic and gravitational energy and assuming the enthalpy,



internal energy, and density function of the pressure  $P$  and the temperature, gives (see Appendix A.2 and Fig. 1.50)

$$\begin{aligned}
 \frac{dU_{\text{tot,C}}}{dt} &= \dot{Q}_C + \dot{W}_C - \dot{m}_{\text{CK}} H_{\text{CK}} \quad \text{for the compression space} \\
 \dot{Q}_C &= 0.0 \\
 \dot{W}_C &= -P \frac{dV_C}{dt} \\
 U_{\text{tot,C}} &= M_C U_C \\
 M_C &= \rho_C V_C \\
 \frac{d}{dt} (\rho_C V_C U_C) + P \frac{dV_C}{dt} &= -\dot{m}_{\text{CK}} H_{\text{CK}}
 \end{aligned} \tag{1.46}$$

$$\begin{aligned}
 \frac{dU_{\text{tot,E}}}{dt} &= \dot{Q}_E - \dot{W}_E + \dot{m}_{\text{HE}} H_{\text{HE}} \quad \text{for the expansion space} \\
 \dot{Q}_E &= 0.0 \\
 \dot{W}_E &= P \frac{dV_E}{dt} \\
 U_{\text{tot,E}} &= M_E U_E \\
 M_E &= \rho_E V_E \\
 \frac{d}{dt} (\rho_E V_E U_E) + P \frac{dV_E}{dt} &= \dot{m}_{\text{HE}} H_{\text{HE}}
 \end{aligned} \tag{1.47}$$

The mass conservation equation applied to the compression and expansion volumes gives (see Appendix A.1 and Fig. 1.50)

$$\begin{aligned}
 \frac{dM_C}{dt} &= -\dot{m}_{\text{CK}} \quad \text{for the compression space} \\
 \frac{d}{dt} (\rho_C V_C) &= -\dot{m}_{\text{CK}} \\
 \rho_C \frac{dV_C}{dt} + V_C \frac{d\rho_C}{dt} &= -\dot{m}_{\text{CK}}
 \end{aligned} \tag{1.48}$$

$$\begin{aligned}
 \frac{dM_E}{dt} &= \dot{m}_{\text{HE}} \quad \text{for the expansion space} \\
 \frac{d}{dt} (\rho_E V_E) &= \dot{m}_{\text{HE}} \\
 \rho_E \frac{dV_E}{dt} + V_E \frac{d\rho_E}{dt} &= \dot{m}_{\text{HE}}
 \end{aligned} \tag{1.49}$$

Combining (1.46) with (1.48), we get

$$\begin{aligned}
 & \frac{dV_C}{dt} (\rho_C U_C + P - \rho_C H_{CK}) + \\
 & \quad \frac{d\rho_C}{dt} V_C (U_C - H_{CK}) + \\
 & \quad \frac{dU_C}{dt} \rho_C V_C = 0 \quad \text{for the compression space} \\
 & \text{with } H_{CK} = H(P, T_{CK}) \\
 & \quad U_C = U(P, T_C) \\
 & \quad \rho_C = \rho(P, T_C)
 \end{aligned} \tag{1.50}$$

In the same way, combining (1.47) and (1.49), we get

$$\begin{aligned}
 & \frac{dV_E}{dt} (\rho_E U_E + P - \rho_E H_{HE}) + \\
 & \quad \frac{d\rho_E}{dt} V_E (U_E - H_{HE}) + \\
 & \quad \frac{dU_E}{dt} \rho_E V_E = 0 \quad \text{for the expansion space} \\
 & \text{with } H_{HE} = H(P, T_{HE}) \\
 & \quad U_E = U(P, T_E) \\
 & \quad \rho_E = \rho(P, T_E)
 \end{aligned} \tag{1.51}$$

Remembering that,

$$\begin{aligned}
 \frac{d\rho}{dt} &= \left( \frac{\partial \rho}{\partial P} \right)_T \frac{dP}{dt} + \left( \frac{\partial \rho}{\partial T} \right)_P \frac{dT}{dt} \\
 \frac{dU}{dt} &= \left( \frac{\partial U}{\partial P} \right)_T \frac{dP}{dt} + \left( \frac{\partial U}{\partial T} \right)_P \frac{dT}{dt}
 \end{aligned}$$

Assuming, for the sake of simplicity in the equations, the following equivalences

$$\begin{aligned}
 (\partial \rho_T)_C &= \left( \frac{\partial \rho_C}{\partial T} \right)_P \\
 (\partial \rho_T)_E &= \left( \frac{\partial \rho_E}{\partial T} \right)_P \\
 (\partial U_T)_C &= \left( \frac{\partial U_C}{\partial T} \right)_P \\
 (\partial U_T)_E &= \left( \frac{\partial U_E}{\partial T} \right)_P
 \end{aligned}$$

and

$$\begin{aligned}
 (\partial \rho_P)_C &= \left( \frac{\partial \rho_C}{\partial P} \right)_T \\
 (\partial \rho_P)_E &= \left( \frac{\partial \rho_E}{\partial P} \right)_T \\
 (\partial U_P)_C &= \left( \frac{\partial U_C}{\partial P} \right)_T \\
 (\partial U_P)_E &= \left( \frac{\partial U_E}{\partial P} \right)_T
 \end{aligned}$$

and defining

$$\begin{aligned}
 \alpha &= \frac{dT_C}{dt} \\
 \beta &= \frac{dT_E}{dt} \\
 \gamma &= \frac{dP}{dt}
 \end{aligned}$$

from (1.50), we get

$$\begin{aligned}
 \alpha &= - \frac{\gamma V_C [(\partial \rho_P)_C (U_C - H_{CK}) + \rho_C (\partial U_P)_C] + \frac{dV_C}{dt} (\rho_C U_C + P - \rho_C H_{CK})}{V_C [(\partial \rho_T)_C (U_C - H_{CK}) + \rho_C (\partial U_T)_C]} \\
 &= \gamma A + B
 \end{aligned} \tag{1.52}$$

from (1.51) we get

$$\begin{aligned}
 \beta &= - \frac{\gamma V_E [(\partial \rho_P)_E (U_E - H_{HE}) + \rho_E (\partial U_P)_E] + \frac{dV_E}{dt} (\rho_E U_E + P - \rho_E H_{HE})}{V_E [(\partial \rho_T)_E (U_E - H_{HE}) + \rho_E (\partial U_T)_E]} \\
 &= \gamma C + D
 \end{aligned} \tag{1.53}$$

We obtain the third differential equation, which defines  $\gamma$ , from the equation of conservation for the total mass contained within the engine (assuming there is no mass leakage)

$$\begin{aligned}
 M_C + M_K + M_R + M_H + M_E &= M_{\text{tot}} \\
 \frac{dM_C}{dt} + \frac{dM_K}{dt} + \frac{dM_R}{dt} + \frac{dM_H}{dt} + \frac{dM_E}{dt} &= 0
 \end{aligned} \tag{1.54}$$

The mass balance equation applied to each single volume gives the respective variations in the mass held within the various volumes

$$\begin{aligned}\frac{dM_K}{dt} &= \dot{m}_{CK} - \dot{m}_{KR} \quad \text{for the cooler} \\ \frac{d}{dt} (V_K \rho_K) &= V_K \frac{d\rho_K}{dt} = \dot{m}_{CK} - \dot{m}_{KR}\end{aligned}\tag{1.55}$$

$$\begin{aligned}\frac{dM_R}{dt} &= \dot{m}_{KR} - \dot{m}_{RH} \quad \text{for the regenerator} \\ \frac{d}{dt} (V_R \rho_R) &= V_R \frac{d\rho_R}{dt} = \dot{m}_{KR} - \dot{m}_{RH}\end{aligned}\tag{1.56}$$

$$\begin{aligned}\frac{dM_H}{dt} &= \dot{m}_{RH} - \dot{m}_{HE} \quad \text{for the heater} \\ \frac{d}{dt} (V_H \rho_H) &= V_H \frac{d\rho_H}{dt} = \dot{m}_{RH} - \dot{m}_{HE}\end{aligned}\tag{1.57}$$

Substituting in (1.54), (1.48), (1.49), (1.55), (1.56), and (1.57), we get (also using (1.52) and (1.53))

$$\begin{aligned}\gamma V_C [(\partial\rho_T)_C A + (\partial\rho_P)_C] + V_C (\partial\rho_T)_C B + \rho_C \frac{dV_C}{dt} + \\ \gamma V_K (\partial\rho_P)_K + \\ \gamma V_R (\partial\rho_P)_R + \\ \gamma V_H (\partial\rho_P)_H + \\ \gamma V_E [(\partial\rho_T)_E C + (\partial\rho_P)_E] + V_E (\partial\rho_T)_E D + \rho_E \frac{dV_E}{dt} = 0\end{aligned}\tag{1.58}$$

Or

$$\gamma = -\frac{1}{E} \left[ \left( V_C (\partial\rho_T)_C B + \rho_C \frac{dV_C}{dt} \right) + \left( V_E (\partial\rho_T)_E D + \rho_E \frac{dV_E}{dt} \right) \right]$$

with

$$\begin{aligned}E &= V_C [(\partial\rho_T)_C A + (\partial\rho_P)_C] + \\ &V_K (\partial\rho_P)_K + \\ &V_R (\partial\rho_P)_R + \\ &V_H (\partial\rho_P)_H + \\ &V_E [(\partial\rho_T)_E C + (\partial\rho_P)_E]\end{aligned}\tag{1.59}$$

To calculate the enthalpy flows, it is indispensable to know the temperatures of the gas crossing the interfaces between the volumes. Let's suppose then that:

- The gas leaves the cooler at temperature  $T_K$ .
- The gas leaves the heater at temperature  $T_H$ .

and that [43]:

$$\text{if } \dot{m}_{CK} > 0 \text{ then } T_{CK} = T_C$$

$$\text{if } \dot{m}_{CK} < 0 \text{ then } T_{CK} = T_K$$

$$\text{if } \dot{m}_{KR} > 0 \text{ then } T_{KR} = T_K$$

$$\text{if } \dot{m}_{KR} < 0 \text{ then } T_{KR} = T_{RK}$$

$$\text{if } \dot{m}_{RH} > 0 \text{ then } T_{RH} = T_{RH}$$

$$\text{if } \dot{m}_{RH} < 0 \text{ then } T_{RH} = T_H$$

$$\text{if } \dot{m}_{HE} > 0 \text{ then } T_{HE} = T_H$$

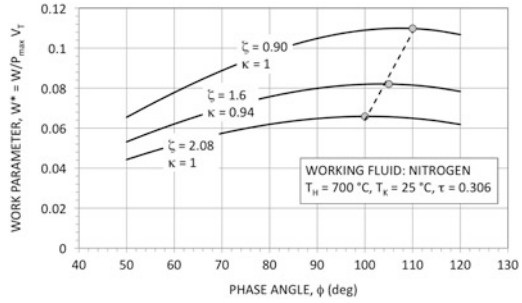
$$\text{if } \dot{m}_{HE} < 0 \text{ then } T_{HE} = T_E$$

If the working fluid is an ideal gas, the specific heat at constant volume  $C_V$  and the specific heat at constant pressure  $C_P$  depend only on the temperature. Consequently, if the regenerator is ideal, the result will be  $T_{RK} \rightarrow T_K$  and  $T_{RH} \rightarrow T_H$ .

The solution of the system of the differential equations (1.52), (1.53), and (1.59) gives the pressure  $P$  in the engine, the temperature  $T_C$  of the gas in the volume of compression and the temperature  $T_E$  of the gas in the volume of expansion. Having fixed the temperature  $T_{RK}$ , the solution is repeated for various  $T_{RH} (\leq T_H)$  until the heat energy  $Q_R$  exchanged globally between the gas and the matrix that constitutes the regenerator, over a complete cycle, is null. The volumetric properties of the working fluid must be calculated by an appropriate equation of state, and we must know the variation in the specific heat  $C_V$  with the temperature under ideal gas conditions (see Sect. 2.4).

The design parameters  $\tau$ ,  $\kappa$ ,  $\zeta$  and  $\phi$  can be varied at will, in practically an infinite number of combinations, provided that the technological limits are respected. The thermodynamic efficiency grows with the ratio  $\tau = T_H/T_K$  (see Sect. 1.1), but the temperatures  $T_H$  and  $T_K$  are restricted by metallurgical limits and by the availability of a cold heat reservoir, respectively. Frequently, the swept volume ratio  $\kappa = V_{SW,C}/V_{SW,E}$  is assumed to be unitary. The optimal number of revs  $N$  is strictly linked to the working fluid and to the power desired: in the ideal case, the useful power increases linearly with the number of revs. Figure 1.51 reports

**Fig. 1.51** Effect of the phase angle  $\phi$  on the work parameter  $W^*$  for three different values of the dead volume ratio  $\zeta$ . The three cases considered correspond to the three engines discussed in [43]

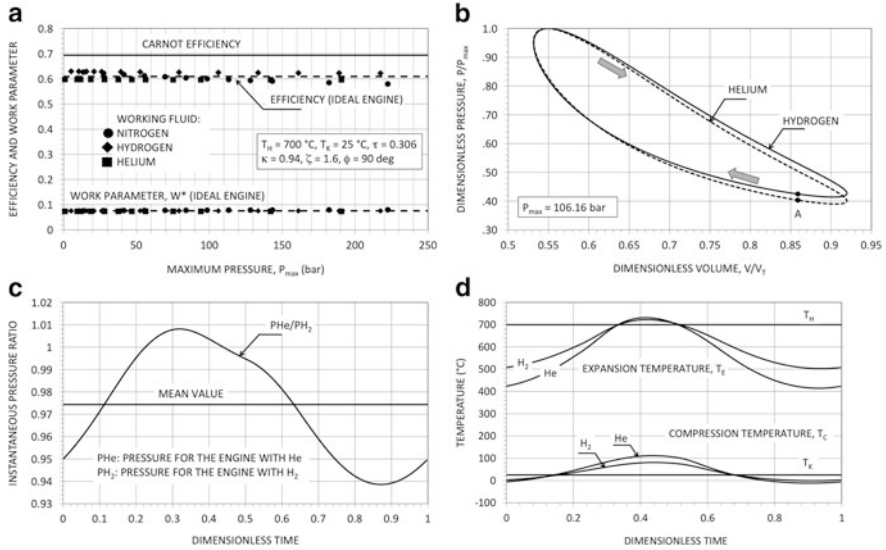


the adimensional work  $W^* = W/P_{\max} V_T$  based on the phase angle  $\phi$  between the compression piston and the expansion piston, for three different values of the parameter  $\zeta = V_D/V_{\text{SW,E}}$ . As the  $\zeta$  ratio increases, so does the dead volume, whilst the pressure variations during the cycle diminish and the useful work produced is reduced. Therefore, the smaller  $\zeta$  is, the greater the useful work. The dead volume, though, should not be null, in order to guarantee the necessary exchanges of thermal energy. There is a value of the  $\phi$  angle which maximises the work parameter  $W^*$  which, in the cases considered in the figure, is about  $100^\circ$ . However, the maximum of  $W^*$  as a function of  $\phi$  is not very pronounced, and even for  $\phi$  values around  $90^\circ$ , we can consider  $W^*$  as being near to its maximum value. Below, we shall always assume  $\phi = 90^\circ$ .

Figure 1.52a shows, as a function of the maximum pressure, the engine efficiency and the work parameter  $W^*$  obtained by applying the adiabatic model to the case of the ideal engine. The geometric parameters assumed are those of the engine GPU-3 described in [43,44].<sup>36</sup> For the three working fluids considered, (nitrogen, hydrogen, and helium) neither the efficiency nor the parameter  $W^*$  varies significantly with the maximum pressure: the mean efficiency value is about 0.61 and the mean value of  $W^*$  is around 0.076. The efficiency of the corresponding ideal Carnot cycle is 0.69 (eight points higher than the value given by the ideal adiabatic analysis).

With helium (monoatomic gas) and hydrogen (diatomic gas) as the working fluids, Fig. 1.52b reports, on the P–V plane, two thermodynamic cycles with the same maximum pressure. Note (see also Fig. 1.52c) that the minimum pressure in the case of the helium engine is lower than the minimum pressure reached using hydrogen. The mean pressure of the helium engine is lower than the hydrogen one and the useful work per cycle is about 3 % lower. The helium in the (adiabatic) volume of expansion cools down significantly more than the hydrogen (see Fig. 1.52d),

<sup>36</sup>The engine GPU-3 (Ground Power Unit) is a rhombic drive Stirling engine generator, developed by General Motors in 1965 for the US Army. The values of the volumes in the engine are as follows:  $V_{\text{CL,C}} = 28.68 \text{ cm}^3$ ,  $V_{\text{CL,E}} = 30.52 \text{ cm}^3$ ,  $V_{\text{SW,C}} = 114.13 \text{ cm}^3$ ,  $V_{\text{SW,E}} = 120.82 \text{ cm}^3$ ,  $V_K = 13.18 \text{ cm}^3$ ,  $V_H = 70.28 \text{ cm}^3$  and  $V_R = 50.55 \text{ cm}^3$ . For the examples considered here, the movement of the pistons is assumed to be pure sinusoidal, in accordance with (1.43a) and (1.43b) with  $\phi = 90^\circ$ .



**Fig. 1.52** Results of the adiabatic analysis for an ideal engine. We have assumed the geometric parameters of the engine GPU-3 (see [43, 44] and the footnote 36). **(a)** Engine efficiency and work parameter for an ideal engine, with different working fluids, based on the maximum pressure. **(b)** Thermodynamic cycle on the plane P–V for two working fluids (helium and hydrogen) at the same maximum pressure. **(c)** Ratio between the instantaneous pressures for two working fluids at maximum pressure  $P_{\max} = 106.16$  bar. **(d)** Trend over time of the gas temperatures in the expansion and compression volumes for two working fluids at the same maximum pressure  $P_{\max} = 106.16$  bar

and, conversely, in the space intended for the compression (adiabatic), it tends to heat up more. It is this which gives us the greater work of compression and the lesser work of expansion in a helium cycle and, as a result, an inferior thermodynamic efficiency (about two points; see Fig. 1.52a).

The real engines give performances (useful power and efficiency) that are completely different from those of ideal engines. The complexity of the thermo-fluid dynamics, which characterises the behaviour of a Stirling engine, means that any detailed analysis of the various losses will be particularly complicated. Here below, the losses will be taken into account only through the introduction of appropriate global parameters. Without any pretensions to absolute precision, we shall consider the efficiency of the regenerator, the compression and the expansion. The pressure losses will not be counted directly as it is assumed that they will contribute anyway to increasing the work of compression and reducing that of expansion.

### *The Thermal Efficiency of the Regenerator*

As we have seen, the regenerator is an essential component of the engine and has to be used if we wish to obtain a high efficiency. Its efficiency  $\epsilon_R$  may be considered,

similarly to (1.35), as the ratio between the heat really transferred to the gas during a complete cycle and the heat transferable to the gas in the case of an ideal regenerator.

Under ideal conditions, the working gas exchanges the maximum possible quantity of heat energy during its passage through the regenerator. In this condition, one of the two temperature differences at the ends of the regenerator,  $(T_{RK} - T_K)$  or  $(T_H - T_{RH})$ , must be zero.

If, on the contrary, the regenerator is not ideal, the minimum temperature difference is different from zero, and, rather than  $\epsilon_R$ , we can assume as our indicator of efficiency the parameter that represents the fractional temperature difference in the regenerator

$$\epsilon = \min(\epsilon_K, \epsilon_H) \quad (1.60)$$

with

$$\epsilon_K = \frac{T_{RK} - T_K}{T_H - T_K} \quad (1.61a)$$

and

$$\epsilon_H = \frac{T_H - T_{RH}}{T_H - T_K} \quad (1.61b)$$

As a result, in the balance equations for the energy and the mass, the temperatures of the gas contained in the regenerator and in the cooler can be assumed, respectively, to be equal to  $T_R = 0.5(T_{RK} + T_{RH})$  and  $T_K = 0.5(T_{RK} + T_K)$ . The temperature of the gas contained in the heater is  $T_H = 0.5(T_{RH} + T_H)$ . In the case of a perfect gas and an ideal regenerator,  $\epsilon = 0$ ,  $T_{RK} = T_{KR} = T_K$ , and  $T_{RH} = T_{HR} = T_H$ .

### *Polytropic Expansion and Compression Efficiencies*

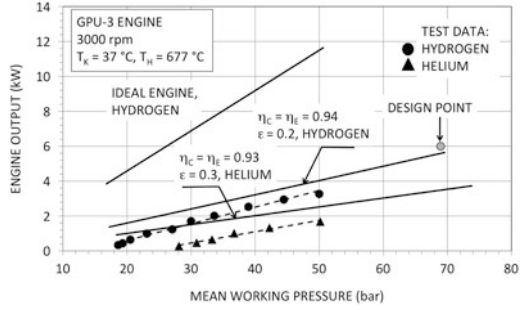
Losses caused by the viscosity of the fluid can be indirectly accounted by means of the compression and expansion efficiencies,  $\eta_E$  and  $\eta_C$ . In an ideal cycle  $\eta_E = \eta_C = 1$ ; in a real process of compression or expansion, in accordance with the volume variation  $dV/dt$ , the work of expansion and of compression will be less and more than the ideal ones, respectively. In this way

$$\dot{W}_C = P \frac{dV_C}{dt} \frac{1}{\eta_C} \quad \text{if} \quad \frac{dV_C}{dt} < 0 \quad (1.62a)$$

$$\dot{W}_C = P \eta_C \frac{dV_C}{dt} \quad \text{if} \quad \frac{dV_C}{dt} > 0 \quad (1.62b)$$



**Fig. 1.53** Power of the GPU-3 engine based on the mean operating pressure of the engine. Comparison between the values calculated and those measured and reported in [44] for hydrogen and for helium



and

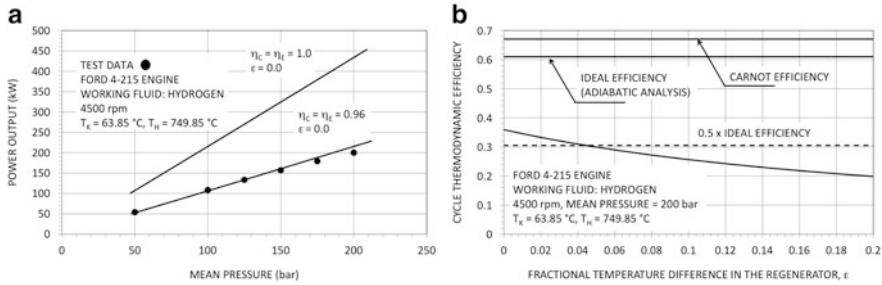
$$\dot{W}_E = P \eta_E \frac{dV_E}{dt} \quad \text{if} \quad \frac{dV_E}{dt} > 0 \quad (1.63a)$$

$$\dot{W}_E = P \frac{dV_E}{dt} \frac{1}{\eta_E} \quad \text{if} \quad \frac{dV_E}{dt} < 0 \quad (1.63b)$$

The useful work during the cycle can be calculated then in the usual way, from the work of compression and expansion.

Figure 1.53 reports the power of the engine GPU-3 as the mean operating pressure varies. The values measured (for hydrogen and helium) are those in [44]. For the calculations, which were done with an adiabatic model, temperatures  $T_K$  and  $T_H$  have been assumed as being  $37^\circ\text{C}$  and  $677^\circ\text{C}$ . The ideal case (see Fig. 1.53) vastly overestimates the useful power. In the case of hydrogen, assuming  $\eta_c = \eta_E = 0.94$  and  $\epsilon = 0.2$ , the real power is described reasonably well: the design power, for example, at the mean pressure of about 70 bar, is 5.64 kW against 6 kW. Testing the same engine with helium gave power at about half of that with hydrogen (at the same mean operating pressure). Assuming  $\eta_c = \eta_E = 0.93$  and  $\epsilon = 0.3$ , there is an acceptable agreement between the values calculated and those that were measured.

The parameters used here in order to take into account the thermodynamic losses of a real engine clearly represent just a rough and largely incomplete description of the real behaviour of an engine. Having chosen the engine and the working fluid, for example, the losses vary with the mean operating pressure and the number of revs. What is more, an engine with a preset geometry, but with different working fluids, is characterised by losses that are quantitatively different, unless operation can be guaranteed with strictly comparable fluid dynamics (as well as its geometry and kinematics). In accordance with this simple model, we need to use different values for the parameters of  $\epsilon$ ,  $\eta_c$  and  $\eta_E$  for the different cases we wish to consider. However, generally speaking, by making the appropriate variations to the number of revs and to the operating pressure, we should get a reasonable similarity in the fluid dynamics. Therefore, having set the reference geometry, we shall assume



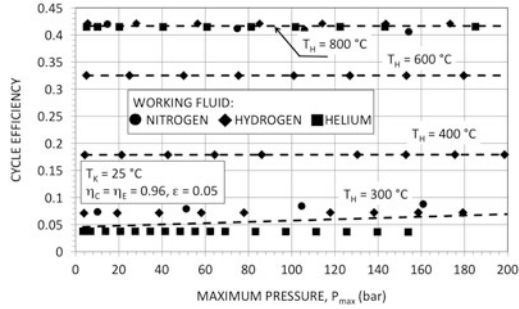
**Fig. 1.54** Various results for the Ford 4-215 engine. The power values have been taken from [43]

constant values for the parameters of  $\epsilon$  and  $\eta_C = \eta_E$  for the various working fluids we consider. The results obtained will be purely indicative of the general thermodynamic behaviour but, for our purposes here, that will be acceptable.

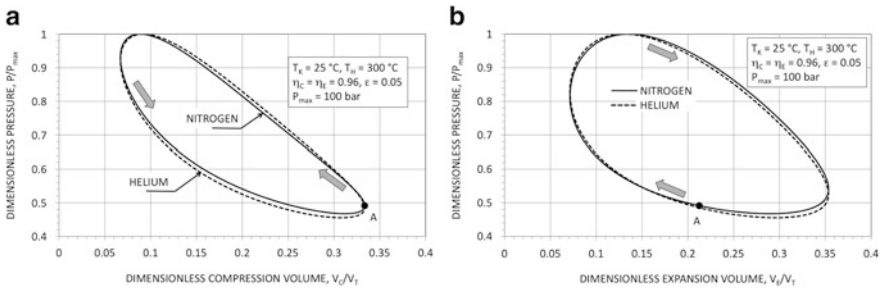
Referring to Fig. 1.53, the engine operating with hydrogen at a mean pressure of 50 bar has a measured efficiency (see [44]) of around 17 %, against a calculated value of about 16 % (obtained by assuming  $\eta_C = \eta_E = 0.94$  and  $\epsilon = 0.2$ ). The efficiency values reported in [44], though, refer to the engine–combustor system and do not deduct the losses associated with the exhaust gases. In [45] the same engine, with helium, at 3,000 rpm, with a mean operating pressure of 69 bar, a maximum temperature of  $650^\circ\text{C}$  and with water for cooling available at  $13^\circ\text{C}$ , the energy from the gases at the combustor outlet was about 34 % of that deriving from combustion. This gave a thermodynamic efficiency of around 35 % (against an efficiency of about 12 % for the entire engine–combustor system on its own). In ref [46, pp. 73, 75] an overall brake thermal efficiency of about 27 % is reported for the same engine (operating with hydrogen, at the temperature extremes of  $735/38^\circ\text{C}$ , with a mean working pressure of 69 bar).

With the purpose of assessing reasonable values for the parameters  $\eta_C$ ,  $\eta_E$  and  $\epsilon$  that relate to the thermodynamic performance of the engine alone, Fig. 1.54a compares several useful power values measured for the Ford 4-215 engine (from [43]), at 4,500 rpm, with the power calculated according to the adiabatic model discussed previously. Assuming  $\eta_C = \eta_E = 0.96$ , the useful power is clearly described, with the variation in the mean pressure of the engine. Figure 1.54b reports the thermodynamic efficiency calculated as the  $\epsilon$  parameter varies for the case with a mean pressure of 200 bar. Where  $\epsilon = 0.05$ , the efficiency is equal to about half the efficiency of the ideal cycle. Assuming such a ratio to be reasonably representative of the quality of the engines made, from now on, we shall assume the efficiencies of compression and expansion to be 0.96 and the fractional temperature difference in the regenerator to be 0.05.

The Stirling engines, operating with a monophasic gas cycle (typically, with the working fluid in the form of an ideal gas and in a similar way to the Brayton cycles), have efficiencies that diminish sharply with the maximum temperature  $T_H$ . For details, see Fig. 1.55. At a maximum temperature of  $800^\circ\text{C}$ , the mean efficiency



**Fig. 1.55** Thermodynamic efficiency of the calculated thermodynamic cycle for different maximum temperatures  $T_H$  based on the maximum operating pressure



**Fig. 1.56** Diagrams P–V for (a) the compression space and (b) the expansion space, for two cycles with helium and nitrogen

(taking the parameters reported in the figure for our calculations) is 58 % of that of a Carnot cycle; at 300 °C the efficiency drops to about 12 % of the ideal maximum. The ratio  $W / (|W_E| + |W_C|)$  between the useful work and the sum of the two works of compression and expansion drops, for example, in the case of hydrogen with  $P_{\max} = 100$  bar, from 0.32 at  $T_H = 800$  °C (with efficiency of 42 %) to 0.047 when  $T_H = 300$  °C (efficiency of 7.2 %).

Figure 1.56a, b shows, in the P–V plane, the variations in pressure as the volume changes for the space of compression and expansion. The case we consider is that with  $T_H = 300$  °C with  $P_{\max} = 100$  bar in Fig. 1.55. The difference in the net work of compression is the reason for the lower efficiency of the helium cycle compared to the cycle with nitrogen as its working fluid. Obviously, such comparisons and results, of a strictly thermodynamic nature, are valid purely within the scope of these extremely simplified hypotheses used for our calculations.

## 1.8.2 Some Final Considerations

Many of the aspects that we have drawn attention to in the closed cycles for gas turbines (Brayton or Joule cycles) are also common to Stirling cycles (consisting of

two isothermal and two isocore transformations, rather than two isentropic and two isobar transformations), which are still not widely used but could have a certain growth in the future, especially in specialist sectors and for small-scale plants (generation supplies, micro-cogeneration), competing with internal combustion engines for these applications.

As we have seen, the fundamental difference from the point of view of the plant, between the gas turbine cycle and the Stirling engine, lies in the fact that the compression and expansion transformations happen in reciprocating machines (a reciprocating piston in a cylinder) rather than turbomachines; however, our considerations regarding fuels, heat exchangers, the presence of the regenerator, the choice of the working fluid, and the regulation are still essentially valid. What is more, the Stirling engine can be adapted to different fuels (natural gas and liquid fuels, but also biomass, solar energy, and waste heat from industrial processes), and it is particularly suitable for the fabrication of low-powered engines (from a few kW to a few dozen kW).

It should be noted, though, that despite the numerous attempts made by some of the greatest manufacturers, dedicating enormous development projects to the Stirling engines—researches which have never failed—the Stirling engine has still not really achieved any commercial success. There are many reasons for this situation, not the least of which being the objective difficulty in successfully competing against the reciprocating engines with internal combustion. Compared to these, the Stirling engine does present certain technological difficulties (in particular, sealing) linked to the presence of working fluids such as helium and hydrogen and to controlling the power.

Despite the objective technological difficulties, there are several engines available today, and most of the companies operating in the sector have focussed on developing engines with an electrical power of a few kW, with costs varying from 5,000 to 10,000 euro/kW.

There is a significant difference, though, in the engine layouts adopted by the producers: alpha, beta, gamma, and dual action alpha configurations, each with their own specific advantages and disadvantages. They also differ in the kind of kinematic mechanism chosen: free piston, rhombic drive, wobble joint, swash plate, rod, and crank.

As mentioned above, several producers are studying solutions that foresee the use of nonconventional fuels, such as biomass or biogas, thereby exploiting the intrinsic flexibility of the Stirling engine with regard to fuel type. Most of the engines developed work at high frequency so as to reduce the specific dimensions of the engine, partly to the detriment of the fluid dynamics and mechanical friction and to component wear, whilst the most commonly used working fluid is nitrogen, given the problems of fluid leakage associated with helium and hydrogen.

## References

1. Dickinson HW (2010) A short history of the steam engine. Cambridge University Press, Cambridge
2. Savery T (1702) THE MINER'S FRIEND; or An Engine to RAISE WATER BY FIRE, DESCRIBED; and of the manner of fixing it in mines; with an account of the several other uses it is applicable unto; and an answer to the objections made against it. Printed for S. Crouch, at the Corner of Pope's Head-alley in Cornhill by W. Clowes, Stamford-street, London. Available via Google. <http://books.google.com/>
3. Callen HB (1985) Thermodynamics and an introduction to thermostatistics, 2nd edn. Wiley, New York
4. Müller-Steinhagen H, Trieb F (2004) Concentrating solar power. A review of the technology. *Ingenia Inform Q R Acad Eng* 18:43–50
5. Macchi E, Campanari S, Silva P (2005) The micro-cogeneration with natural gas. Polipress, Polytechnic of Milano, Milano (in Italian)
6. Romano M (2012) The thermoelectric plants. In: Pedrocchi E, Alimonti G (eds) Energy, environmental, and development. Progetto Leonardo, Esculapio, Bologna (in Italian)
7. Ortolani C (1984) Analysis of a modern thermoelectric plant of 300 MW. Clup, Milano (in Italian)
8. Visser WPJ, Shakariyants SA, Oostveen M (April 2011) Development of a 3 kW microturbine for CHP applications. *J Eng Gas Turb Power* 133:042301-1–042301-8
9. Macchi E, Pelló PM, Sacchi E (1984) Cogeneration and district heating. Thermodynamic and economical aspects. Clup, Milano (in Italian)
10. Anon (January 1985) Definitions and classification of the cogeneration processes. Draft Norm. Progetto CTI–11/146 a E02.11.146.1, Comitato Termotecnico Italiano, Milano (in Italian)
11. Zanelli F, Bonomo A, Beretta GP (2000) Fuel savings and reduction of greenhouse gases in a large waste-to-energy cogeneration facility. In: Proceedings of the 35th intersociety energy conversion engineering conference, paper AIAA-00-3059, pp 1434–1442
12. Anon (December 1999) Cogeneration total energy module - TOTEM - AdvanceD - Base 15 kW. Use and maintenance manual. TemEnergy Cogeneration Systems, Nosate (Milano)
13. Tastavi A (1990) TOTEM. Technical report. Combustion gas analysis of the cogeneration group TOTEM with and without a catalytic converter supplied with natural gas or biogas. LENI-Report-1990-001
14. Macchi E (1983) Prime movers for vapour compression heat pumps. In: Berghmans J (ed) Heat pump fundamentals. Proceedings of the NATO advanced study institute on heat pump fundamentals, Espinho, 1–12 September 1980. Martinus Nijhoff Publishers, The Hague, pp 192–225
15. Smil V (2010) Two prime movers of globalization. The history and impact of diesel engines and gas turbines. The MIT Press, Cambridge
16. Pagliano L (1989) Thermodynamic rationalization of the production of low-temperature heat. PhD Thesis, Polytechnic of Milan
17. Leyzerovich AS (May 2007) Steam turbines: how big can they get? *Mod Power Syst* 27(5): 43–50
18. Twardowsky A (October 2007) 858 MW supercritical extension for Belchatow. *Mod Power Syst* 27(10):12–17
19. Leyzerovich AS (December 2009) Supercritical-pressure power plants: a progress report. *Mod Power Syst* 29(12):31–39
20. Kjaer S, Bugge J, Stolzenberger C (2004) Europeans still aiming for 700 °C steam. *Mod Power Syst* 24(11):19–25
21. Jourdain V, Herbaut P (2010) Full steam ahead for Flamanville 3 EPR turbine island construction. *Mod Power Syst* 30(5):17–25
22. Lozza G (2007) Gas turbine and combined cycles, 2nd edn. Progetto Leonardo, Bologna (in Italian)

23. Rohlik HE, Kofskey MG, Katsanis T (1967) Summary of NASA radial turbine research related to Brayton cycle space power systems. In: *Advances in energy conversion engineering. Intersociety energy conversion engineering conference*, Miami Beach, Florida, 13–17 August 1967, pp 45–54
24. Takizuka T, Takada S, Yan X, Kosugiyama S, Katanishi S, Kunitomi K (2004) R&D on the power conversion system for gas turbine high temperature reactors. *Nucl Eng Des* 233:329–346
25. Thomas S (2011) The pebble bed modular reactor: an obituary. *Energ Pol* 39:2, 431–432, 440
26. Kays WM, London AL (1998) *Compact heat exchangers*, 3rd edn. Krieger Publishing Company, Malabar
27. Kays WM, Crawford ME (1993) *Convective heat and mass transfer*, 3rd edn. McGraw-Hill, New York
28. El-Wakil MM (1962) *Nuclear power engineering*. McGraw-Hill, New York
29. Barret MJ (2003) Performance expectations of closed-Brayton-cycle heat exchangers in 100 kW nuclear space power systems. In: *First international energy conversion engineering conference*. Portsmouth, Virginia, Paper No. AIAA-2003-5956, 17–21 August 2003
30. Invernizzi CM, Iora P, Sandrini R (2011) Biomass combined cycles based on externally fired gas turbines and organic Rankine expanders. *Proc IME J Power Eng* 225:1066–1075
31. Lee JC, Campbell J Jr, Wright SE (1981) Closed-cycle gas turbine working fluids. *J Eng Power Trans ASME* 103:220–228
32. Turton RK (1995) *Principles of turbomachinery*, 2nd edn. Chapman & Hall, London
33. Fruttschi H U (2005) *Closed-cycle gas turbines. Operating experience and future potential*. ASME Press, New York
34. McDonald CF (2012) Helium turbomachinery operating experience from gas turbine power plants and test facilities. *Appl Therm Eng* 44:108–142
35. McCormick JE, Redding TE (1967) 3 kW Recuperated closed Brayton-cycle electrical power system. In: *Advances in energy conversion engineering Intersociety energy conversion engineering conference*, Miami Beach, Florida, 13–17 August 1967, pp. 1–7
36. Anon (1951) Philips air-engine generator MP 1002 CA. Directions for use. Royal Philips Electronics
37. Ross B, Dudenhoefer JE (1991) Stirling machine operating experience. In: *26th intersociety energy conversion engineering conference*, Boston, 4–9 August 1991
38. Hawley JG, Ashcroft SJ, Patrick MA (1994) Advanced underwater power systems. *Proc IME J Power Eng* 208:37–45
39. Senft JR (1993) *Ringbom stirling engines*. Oxford University Press, New York
40. Finkelstein T, Organ AJ (2001) *Air engines*. ASME Press, The American Society of Mechanical Engineers, New York
41. Finkelstein T (1960) Generalized thermodynamic analysis of Stirling engine. SAE Paper, No. 118B
42. Walker G (1980) *Stirling engines*. Clarendon Press/Oxford University Press, Oxford/New York
43. Urieli I, Berchowitz DM (1984) *Stirling cycle engine analysis*. Adam Hilger Ltd, Techno House, Redcliffe Way, Bristol
44. Cairelli JE, Thieme LG, Walter RJ (1978) Initial test results with a single-cylinder rhombic-drive Stirling engine. DOE/NASA/1040-78/1, NASA TM-78919
45. Thieme LG (1979) Low-power baseline test results for the GPU 3 Stirling engine. DOE/NASA/1040-79/6, NASA TM-79103
46. Percival WH (1974) Historical review of Stirling engine development in the United States from 1960 to 1970. NASA CR-121097
47. Casci C (1978) *Elements of fluid machines - two phase fluid machines*. Masson Italia Editori, Milan (in Italian)
48. Bombarda P (2010) Some notes on the heat exchangers - characteristics and performance expectations. Polytechnic of Milan (in Italian)